Old and New on eigenvalues of the Schur complement of the Stokes operator.

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The lowest eigenvalue of the Schur complement of the Stokes operator coincides with the square root of the inf-sup constant of the divergence, also called LBB constant (after Ladyzhenskaya, Babuska and Brezzi). This quantity receives much attention in the literature. Most available results are for two dimensional domains. In a widely know paper [1], Horgan & Payne relate this constant with the Friedrichs constant and deduce a simple lower bound for the LBB constant. This lower bound was popularized as the Horgan & Payne angle.

One year ago, we discovered that the proof of H&P is flawed. A few weeks ago we found a counter-example. In this talk I will tell this story, and also some positive results we were able to prove, improving a recent result by Duran [2] for two-dimensional domains.

[1] C.O. Horgan, L.E. Payne – On inequalities of Korn, Friedrichs and Babuska–Aziz, Arch. Rat. Mech. Anal., 82 (1983), 165–179.

[2] R.G. Duran – An elementary proof of the continuity from $L^2_0(\Omega)$ to $H^1_0(\Omega)^n$ of Bogovskiis right inverse of the divergence. Prepublication (2010), 19p.