Active Vibration Control by the Receptance Method

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Research Record

BSc (Iran 1999-2002)

“Vibration Analysis of an Asymmetric Aerofoil Cross-section Turbine blade Using Galerkin’s Method”

MSc (Liverpool 2003-2004)

“Vibration Suppression Using Vincent’s Circle”

PhD (Liverpool 2004-2007)

“Passive Modification and Active Vibration Control Using the Receptance Method”

Postdoctoral Research (Liverpool 2007-2009)

EPSRC Grant ER/F008724/1

“ A New Approach on Active Vibration Suppression”

Temporary Lectureship (Liverpool 2009-2010)

Lectureship (Southampton 2010-present)

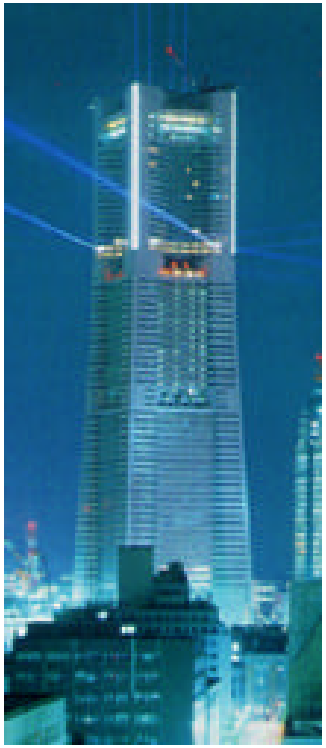
Introduction

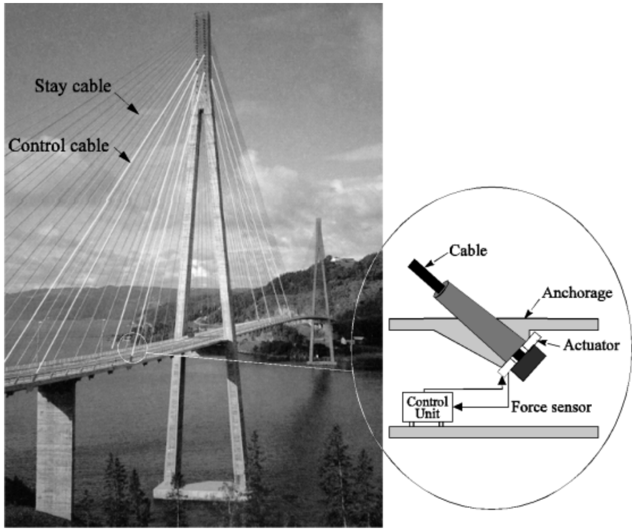
- Background
- Definition of Receptance
- Active Vibration Control by the Receptance Method
- Output Feedback
- State feedback
- Eigenvalue Sensitivity Analysis
- Partial Pole Placement
- Robust Pole Placement
- Active control on Agusta-Westland helicopter W30
- Future research
- Conclusions

Active Vibration Control of Structures

Civil
Automotive
Aerospace

Active damping bridges between towers





Active tendon control for bridges

Active Vibration Control

- **Limitations in passive modification**
  - Difficulty in measuring the rotational degrees of freedom
  - Large and inaccurate structural models (stiffness, mass and damping)
  - Limitation in the form of modification (Symmetry, positive-definite)
  - The rank of modification

Introduction to Receptance Method

Inverse of the dynamic stiffness matrix,

$$H(s) = \left(s^2 M + sC + K\right)^{-1}$$

The transfer function between input and output data,

$$x(s) = H(s) f(s)$$

The ratio of two polynomials,

$$H(s) = \frac{N(s)}{d(s)}$$

In terms of the eigenvalues and eigenvectors,

$$H(s) = \sum_{k=1}^{n} \left(\frac{\varphi_k \varphi_k^T}{\left(s - \lambda_k\right)} + \frac{\varphi_k^{\text{¿}} \varphi_k^{\text{¿}\,T}}{\left(s - \lambda_k^{\text{¿}}\right)} \right)$$

Active Vibration Control by the Receptance Method

- There is no requirement to know or to evaluate the $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ matrices.

- The receptance equations $x(s)=H(s)f(s)$ are made complete with a small number of measured force inputs.

- There is no requirement for an observer or for model reduction.

- The method is general and can be applied to any input-output measured data.

Practical Application of Active Vibration Control by the Receptance Method

- Control forces are applied using actuators (piezo devices, proof-mass actuators, electro-hydraulic etc) with power amplifiers. These have dynamics which must be modelled by conventional (matrix) methods.
- Responses are measured using sensors (piezo-strip devices, ICP accelerometers etc). These also have dynamic behavior that must be modelled by the conventional approach.
- Modelling of actuators and sensors is unnecessary by the receptance method. We simply generalise it to the frequency response function between any input and any output – typically voltage input to the power amplifier, voltage output from an ICP device.
- The measured open-loop FRF is a complete model of the systems, including any time delays due to A/D, D/A conversion, integration of accelerometer signals etc.

Output Feedback Control

1. Mathematical model of the system

$$\left(s^2 M + sC + K\right) x(s) = \mathbf{Bu}(s) + p(s)$$

$$y(s) = \mathbf{Dx}(s)$$

2. Control Law

$$u(s) = -\left(G + sF\right) y(s)$$

3. Collocated sensor and actuator

$$D = B^T \in \Re^{m \times n}$$

$$\left(s^2 M + sC + K + B\, diag\left(g_i + sf_i\right) B^T\right) x(s) = p(s)$$

Pole - Zero Assignment

$$H(s) = \left(s^2 M + sC + K\right)^{-1}$$ Receptance Matrix

$$\left(s^2 M + sC + K + B\, diag\left(g_i + sf_i\right) B^T\right) x(s) = p(s)$$

$$\left(I + H(s)\, \mathbf{\Delta Z}(s)\right) x(s) = H(s)\, p(s)$$

$$\mathbf{\Delta Z}(s) = B\, diag\left(g_i + sf_i\right) B^T$$ Active modification

$$\boxed{x(s) = \left(I + H(s)\, B\, diag\left(g_i + sf_i\right) B^T\right)^{-1} H(s)\, p(s)}$$

$$x(s)=\frac{adj\left(I+H(s)B\,diag\left(g_i+sf_i\right)B^T\right)}{\det\left(I+H(s)B\,diag\left(g_i+sf_i\right)B^T\right)}H(s)\,p(s)$$

Pole assignment

Natural frequencies given from the denominator characteristic equation.

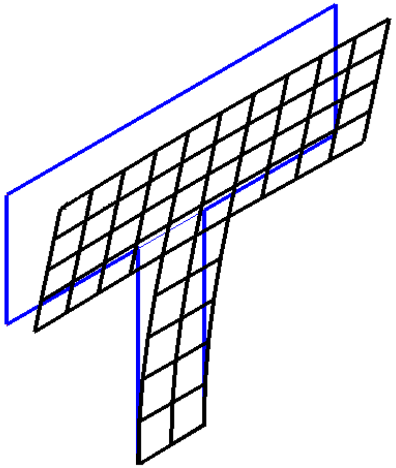
$$\det\left(I+H\left(\lambda_j\right)\,B\;diag\left(g_i+\lambda_j f_i\right)\,B^T\right)=0\,;\quad j=1,\,2,\,\ldots,\,r\,;\quad r\leq 2\mathrm{n}$$

Zero assignment

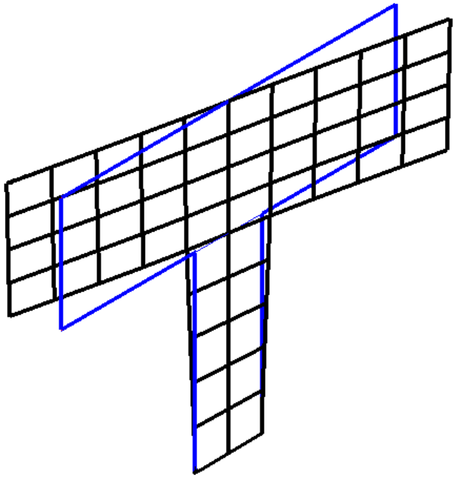
Antiresonances given by the zeros of the numerator matrix terms –The antiresonances are
generally different for the different receptance terms.

$$\left[adj\left(I+H(\mu_k)B\,diag\left(g_i+\mu_k f_i\right)B^T\right)H(\mu_k)\right]_{pp}=0$$

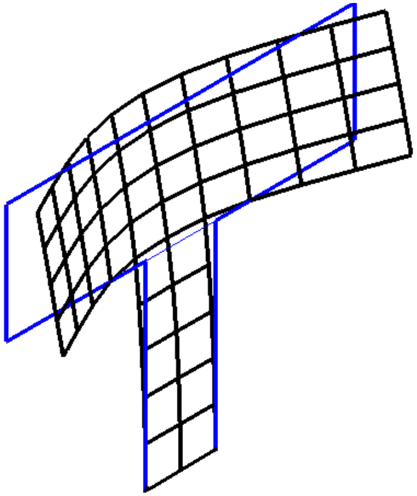
Active Vibration Control of the T-Plate
Output Feedback



1st mode: 42Hz

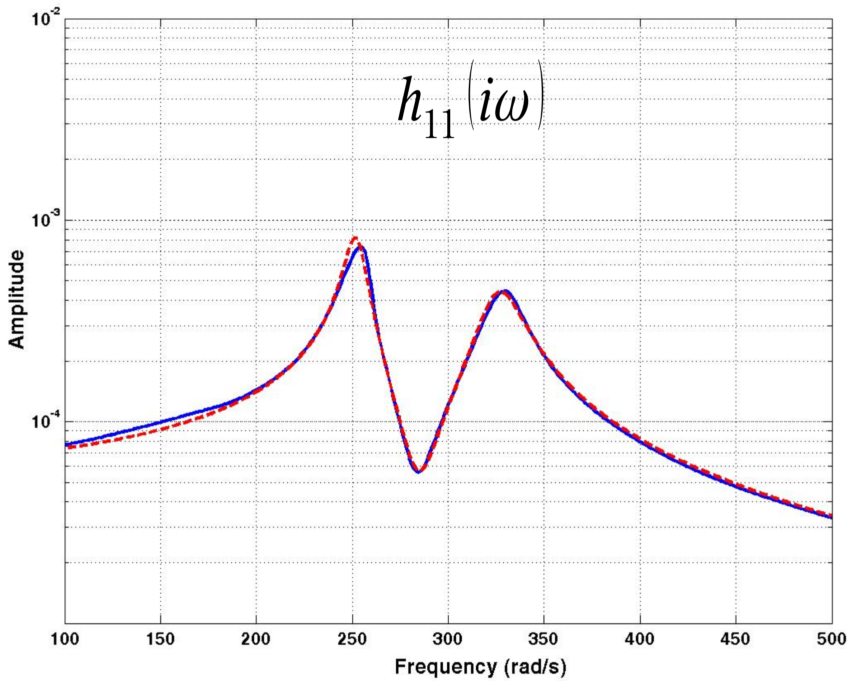


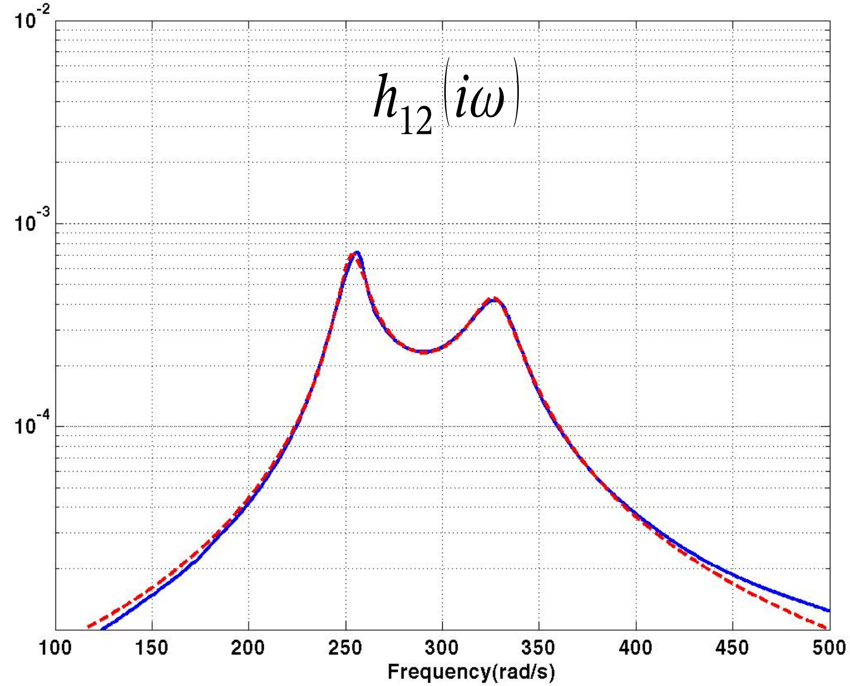
2nd mode: 53Hz



3rd mode: 130Hz

Rational Fraction Polynomials

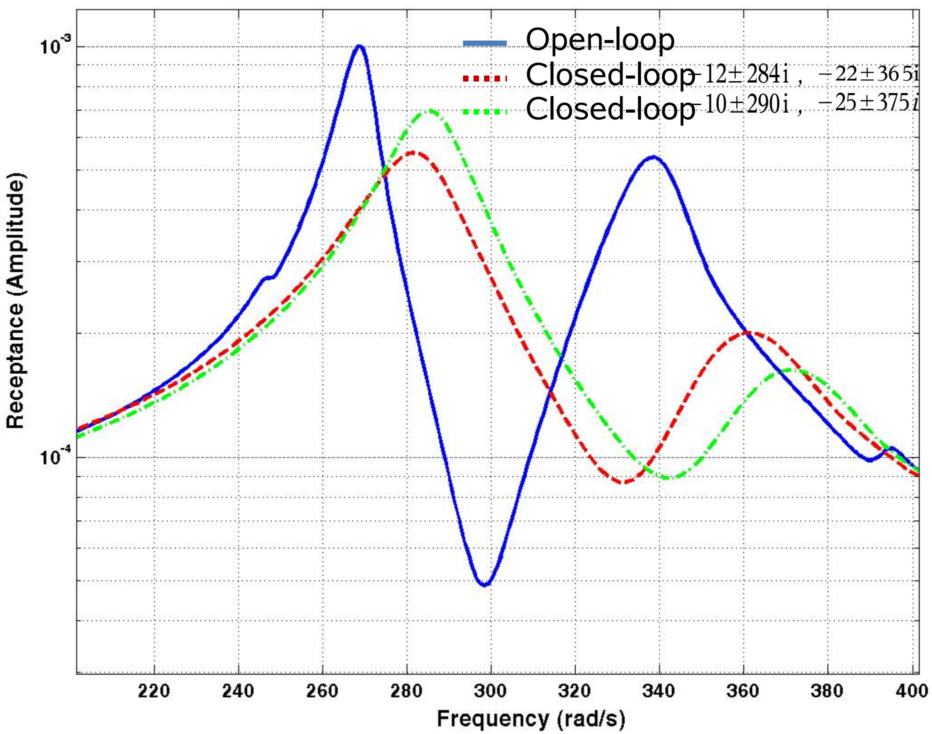


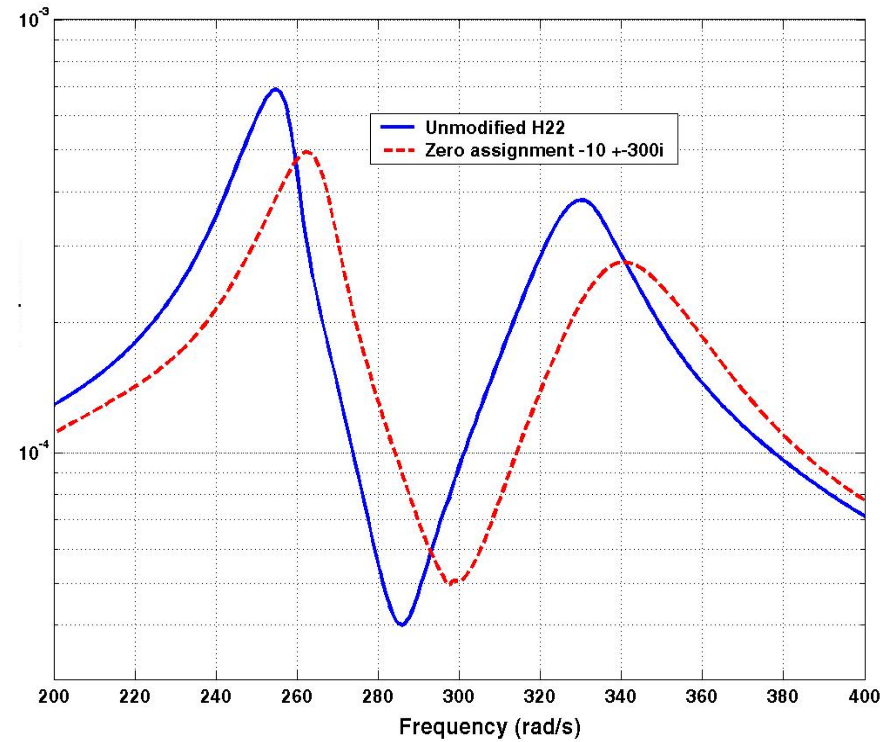


$$h_{11}(s)=h_{22}(s)=\frac{7.956\times10^{-10}s^2+1.382\times10^{-8}s+6.438\times10^{-5}}{1.476\times10^{-10}s^4+4.987\times10^{-9}s^3+2.515\times10^{-5}s^2+0.0003862s+1}$$

$$h_{12}(s)=h_{21}(s)=\frac{-1.334\times10^{-10}s^2-4.678\times10^{-9}s+5.208\times10^{-6}}{1.451\times10^{-10}s^4+5.034\times10^{-9}s^3+2.494\times10^{-5}s^2+0.0003999s+1}$$

Pole and Zero Assignment





J.E. Mottershead, **M.G. Tehrani**, S. James and Y.M. Ram, Active vibration suppression by pole-zero placement using measured receptances, *Journal of Sound and Vibration*, 311(3-5), 2008, 1391-1408.

Single-Input State Feedback

$$M\ddot{x}+C\dot{x}+\mathbf{Kx}=bu(t)+p(t)$$

Control Force

$$u(t)=-f^{T}\dot{x}-g^{T}x$$

$$\left(s^{2}M+s\left(C+\mathbf{bf}^{T}\right)+\left(K+\mathbf{bg}^{T}\right)\right)x(s)=p(s)$$

Rank-1 Modification

State feedback

Sherman-Morrison Formula (Rank 1 modification)

$$\hat{A}(s) = A(s) + \mathbf{uv}^{\boldsymbol{T}}$$

$$\hat{A}^{-1}(s) = A^{-1}(s) - \frac{A^{-1}(s)\,\mathbf{uv}^{\boldsymbol{T}} A^{-1}(s)}{1 + v^T A^{-1}(s)\,u}$$

$$A^{-1} = H \qquad u = b \qquad v = (g + sf)$$

Single input state feedback

$$\hat{H}(s) = H(s) - \frac{H(s)\,b\,(g + sf)^T H(s)}{1 + (g + sf)^T H(s)\,b}$$

State Feedback Theory

Given: $\left(s^2M+sC+K\right)x(s)=\mathrm{B}u(s)+p(s)$, and a complex set $\left\{\mu_1 \quad \mu_2 \quad \cdots \quad \mu_{2n}\right\}$ closed under conjugation.

Find: $g\in R^n \quad f\in R^n$ such that $\left(g+\mu_k f\right)^T H\left(\mu_k\right) b=-1$

for $k=1, 2,\dots, 2n$

Solution:

Denote $r_k=H\left(\mu_k\right)b$

Then we need to solve

$$r_k^T g+\mu_k r_k^T f=-1 \qquad k=1,2,\dots,2n$$

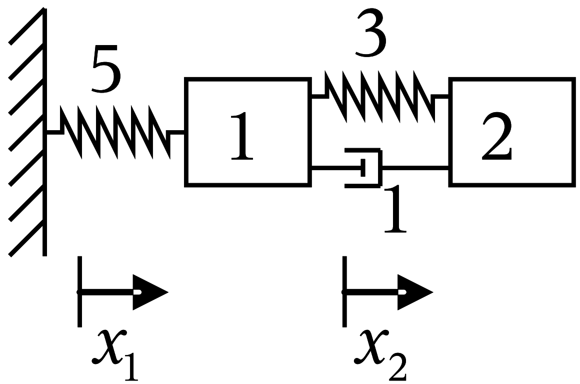
$$\begin{bmatrix} r_1^T & \mu_1 r_1^T \\ r_2^T & \mu_2 r_2^T \\ \vdots & \vdots \\ r_{2n}^T & \mu_{2n} r_{2n}^T \end{bmatrix} \begin{pmatrix} g \\ f \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ \vdots \\ -1 \end{pmatrix} \qquad G = \begin{bmatrix} r_1^T & \mu_1 r_1^T \\ r_2^T & \mu_2 r_2^T \\ \vdots & \vdots \\ r_{2n}^T & \mu_{2n} r_{2n}^T \end{bmatrix}$$

1. $\mathbf{G}$ is invertible if the system is controllable and $\mu_1, \mu_2, \ldots \mu_{2n}$ are distinct.

3. If $\mathbf{G}$ is invertible and the set $\mu_1, \mu_2, \ldots \mu_{2n}$ is closed under conjugation then $\mathbf{g}$ and $\mathbf{f}$ are real.

5. In principle the 2n poles can be assigned using a single input.

Example



$$M=\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix} \qquad C=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} \qquad K=\begin{bmatrix}8 & -3\\ -3 & 3\end{bmatrix}$$

With $\mathbf{b}=(1 \quad 2)^T$ we wish to assign poles to:

$$\mu_1=-1+10\,i \qquad \mu_2=-1-10\,i \qquad \mu_3=-2 \qquad \mu_4=-3$$

$$\boldsymbol{r}_k^T \boldsymbol{g} + \boldsymbol{\mu}_k \boldsymbol{r}_k^T \boldsymbol{f} = -1$$

$$H(\mu_2)\,b = \left((-1-10i)^2 M + (-1-10i)\,C + K\right)^{-1} b = \begin{pmatrix} -0.0102-0.0021i \\ -0.0097-0.0020i \end{pmatrix}$$

$$H(\mu_3)\,b = \begin{pmatrix} 0.1236 \\ 0.2360 \end{pmatrix}$$

$$H(\mu_4)\,b = \begin{pmatrix} 0.0714 \\ 0.1111 \end{pmatrix}$$

$$\begin{bmatrix} -0.0102+0.0021i & -0.0097+0.0020i & -0.0110-0.1043i & -0.0101-0.0989i \\ -0.0102-0.0021i & -0.0097-0.0020i & -0.0110+0.1043i & -0.0101+0.0989i \\ 0.1236 & 0.2360 & -0.2472 & -0.4719 \\ 0.0714 & 0.1111 & -0.2143 & -0.3333 \end{bmatrix} \begin{pmatrix} g \\ f \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ -1 \\ -1 \end{pmatrix}$$

Solution:

$$g = [68.8750 \;\; 30.3750]^T \qquad f = [-62.6750 \;\; 68.1750]^T$$

Solve the eigenvalue problem:

$$\det\left(A-\lambda B\right)=0$$

$$A=\begin{bmatrix} 0 & I \\ -\left(K+\mathbf{b}\mathbf{g}^{T}\right) & -\left(C+\mathbf{b}\mathbf{f}^{T}\right) \end{bmatrix} \qquad B=\begin{bmatrix} I & 0 \\ 0 & M \end{bmatrix}$$

As requested, the eigenvalues are:

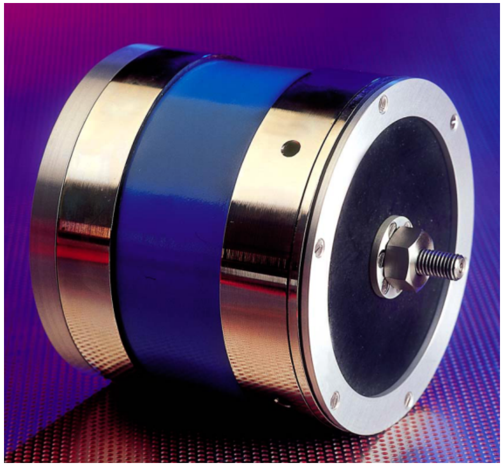
$$\mu_1=-1+10i$$

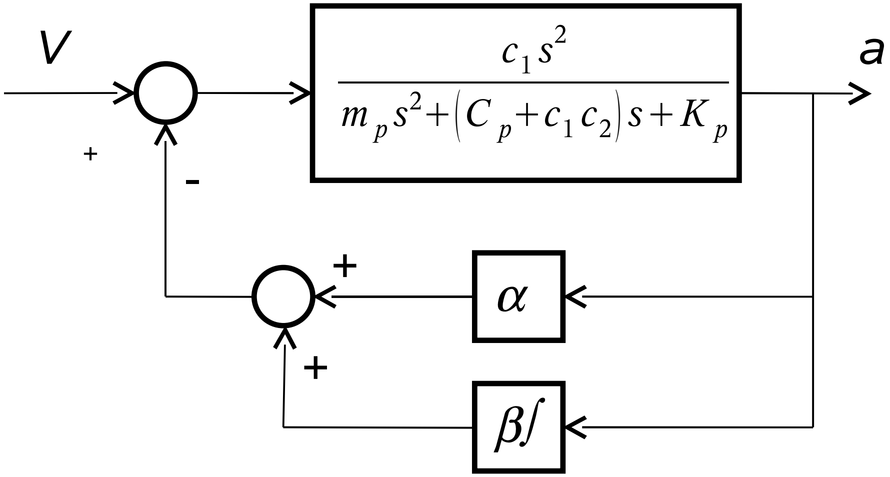
$$\mu_2=-1-10\,i$$

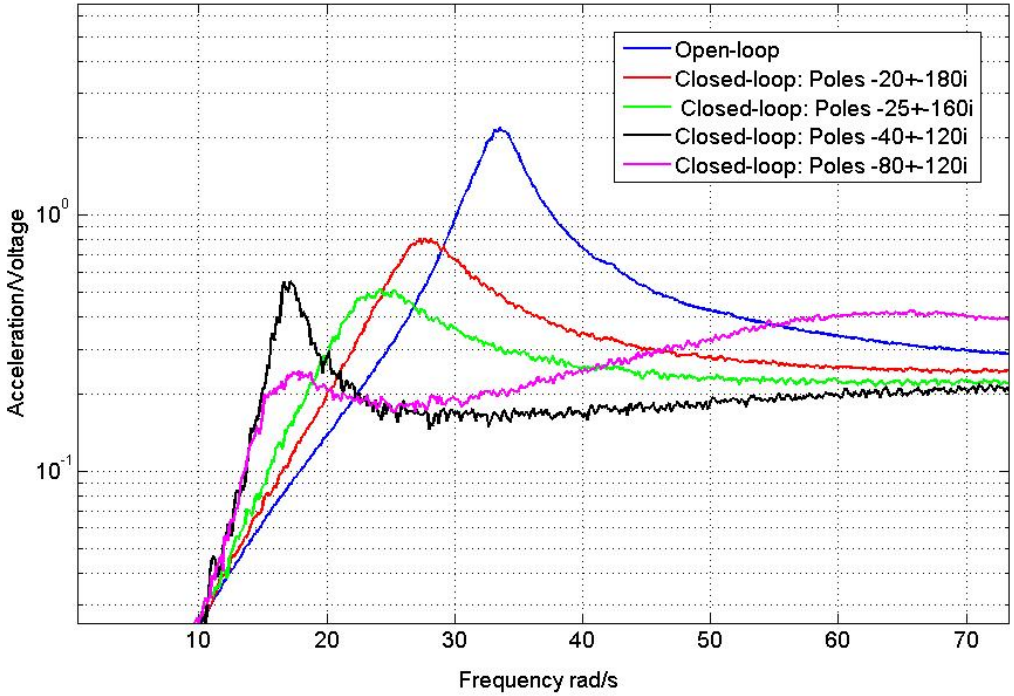
$$H\left(s\right)=\left(s^2M+sC+K\right)^{-1}$$

$$\mu_4=-3$$

Control of Actuator Poles

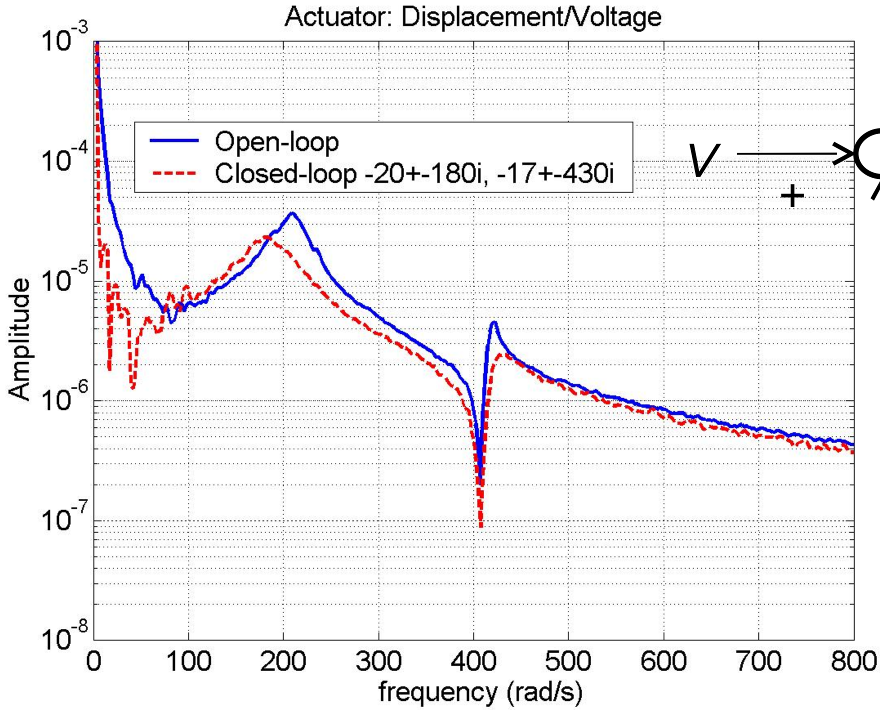


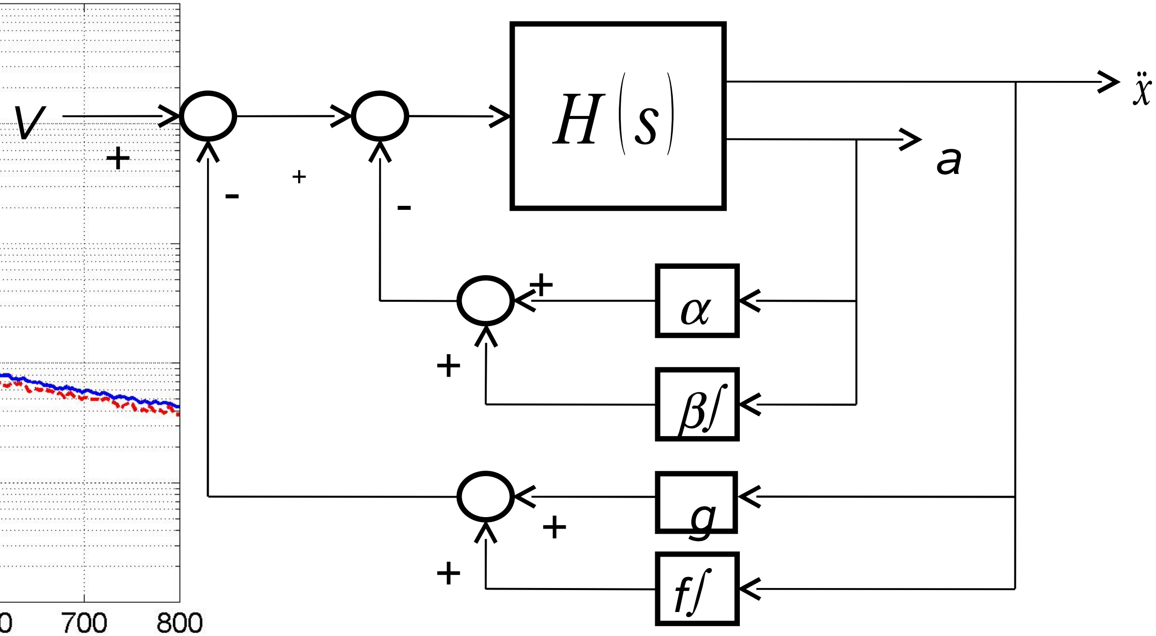




State Feedback

Short Beam Experiment





Sensitivity Analysis

Characteristic equations: $$1+\left(g+\mu_j f\right)^T \frac{N\left(\mu_j\right)}{d\left(\mu_j\right)} b=0$$

Perturbation due to a small change in the control gains:

$$N\left(\mu_j+\delta\mu_j\right)=N\left(\mu_j\right)+\frac{\partial N}{\partial s}\Big|_{s=\mu_j}\frac{\partial \mu_j}{\partial g}\delta g$$

$$d\left(\mu_j+\delta\mu_j\right)=d\left(\mu_j\right)+\frac{\partial d}{\partial s}\Big|_{s=\mu_j}\frac{\partial \mu_j}{\partial g}\delta g$$

Results in linear equations in the control gains:

$$S_{ji}^{g}=\frac{\partial \mu_j}{\partial g_i}=\frac{-e_i^T N\left(\mu_j\right) b}{\frac{\partial d}{\partial s}\Big|_{s=\mu_j}+\left(g+\mu_j f\right)^T \frac{\partial N}{\partial s}\Big|_{s=\mu_j} b+f^T N\left(\mu_j\right) b}$$

$$S_{ji}^{f}=\mu_j S_{ji}^{g}$$

Sensitivity Analysis

- Assignment of the eigenvalue sensitivities.

- Development of the sensitivity equations with respect to the changes in the control gains.

- Development of the sensitivity equations with respect to the errors in the measured receptance terms.

- Partial pole placement using the sensitivity analysis.

J. E. Mottershead, **M. G. Tehrani** and Y. M. Ram, Assignment of eigenvalue sensitivities from receptance measurements, *Mechanical Systems and Signal Processing*, 23, 2009,1931-1939.

Partial Pole Placement

Partial pole placement is the problem of assigning certain poles, while keeping the other poles of interest unchanged.

$$\varphi_k^T\left(M\lambda_k^2+C\lambda_k+K\right)\varphi_k=-\left(\varphi_k^T b\right)\left(\left(g^T+\lambda_k f^T\right)\varphi_k\right)$$

Uncontrollability condition:

$$\left(\varphi_k^T b\right)=0 \qquad\qquad b=\mathrm{null}\left(\varphi_k^T\right)$$

Numerical Example

$$M=\begin{bmatrix} 3 & & & \\ & 10 & & \\ & & 20 & \\ & & & 12 \end{bmatrix} \quad C=\begin{bmatrix} 2.3 & -1 & & \\ -1 & 2.2 & -1.2 & \\ & -1.2 & 2.7 & -1.5 \\ & & -1.5 & 1.5 \end{bmatrix} \quad K=\begin{bmatrix} 40 & -30 & & \\ -30 & 60 & -30 & \\ & -30 & 90 & -30 \\ & & -30 & 30 \end{bmatrix}$$

The open-loop poles :

$$\lambda_{1,2}=-0.0108\pm0.8736i$$
$$\lambda_{3,4}=-0.0809\pm1.6766i$$
$$\lambda_{5,6}=-0.1336\pm2.5280i$$
$$\lambda_{7,8}=-0.3980\pm4.0208i$$

We wish to assign the first two pairs of poles while the remaining poles are unchanged.

$$\mu_{1,2}=-0.03\pm1i$$
$$\mu_{3,4}=-0.1\pm2i$$

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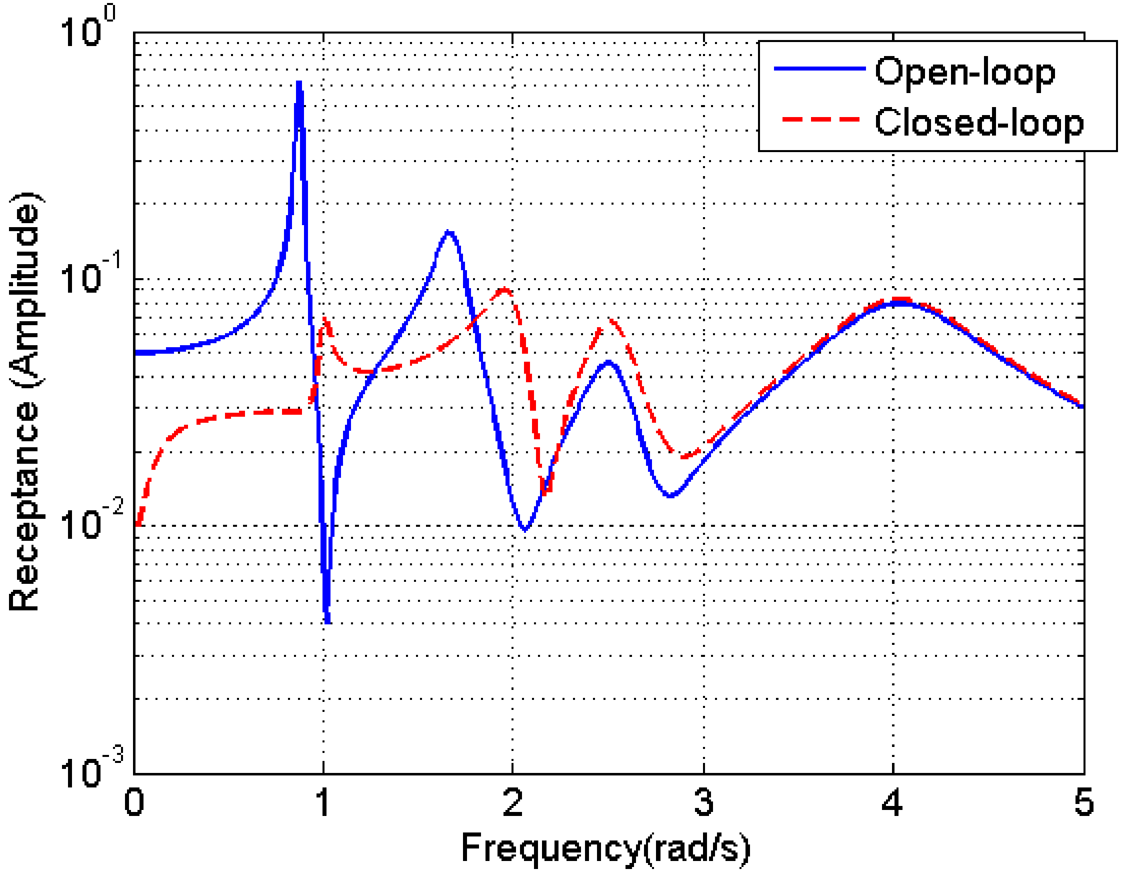
Partial Pole Placement

$$b(s)=b_1+\frac{b_2}{s}$$

$$\begin{bmatrix} \varphi_5^T & \dfrac{\varphi_5^T}{\lambda_5} \\ \varphi_7^T & \dfrac{\varphi_7^T}{\lambda_7} \end{bmatrix}\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}=0$$

$$b=\begin{pmatrix} 0.1996 \\ 0.7146 \\ 0.2723 \\ -0.5712 \end{pmatrix}+\frac{1}{s}\begin{pmatrix} 0.1143 \\ 0.1512 \\ 0.0898 \\ -0.0719 \end{pmatrix}$$

$$g=\begin{pmatrix} 10.1513 \\ 12.1105 \\ 8.1401 \\ 7.2688 \end{pmatrix} \quad f=\begin{pmatrix} 4.4973 \\ 5.4989 \\ 6.4880 \\ 11.2309 \end{pmatrix}$$



Experiments: General Procedure

- Measure the open loop input-output FRF over a desired frequency range.

- Fit MIMO rational fraction polynomials to the measure FRF and obtain the input-output transfer function.

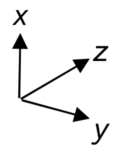
- Select the force distribution vector $\mathbf{b}(s)$ – possibly for partial pole placement.

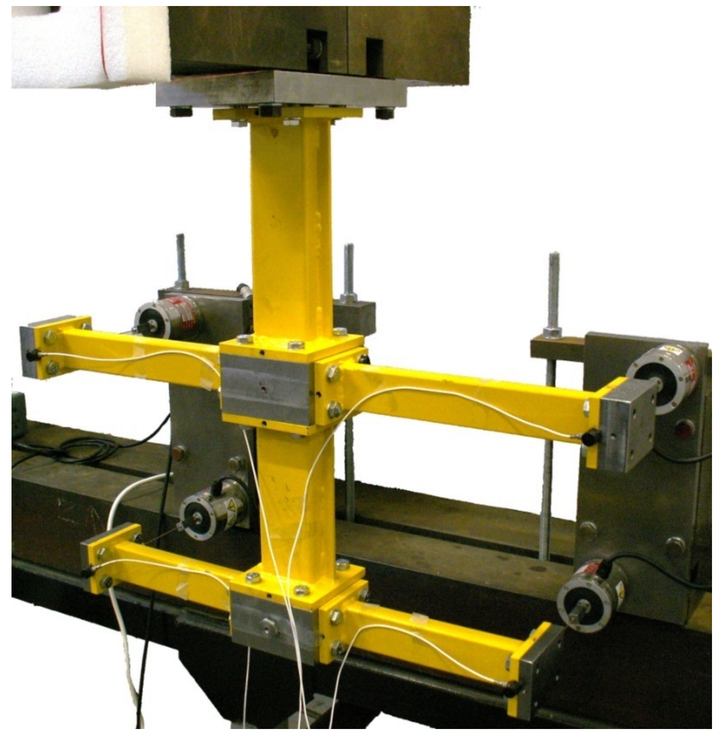
- Apply the Sherman-Morrison formula to obtain characteristic equations in the unknown gains, $\mathbf{g}$, $\mathbf{f}$.

- In the case of robust pole placement, minimise the sensitivity to measurement error, subject to, $[G]\begin{pmatrix} g \\ f \end{pmatrix} = \begin{pmatrix} -1 \\ \vdots \\ -1 \end{pmatrix}$

- Implementation of the controller using dSPACE in real time.

Partial Pole Placement
Modular Test Structure

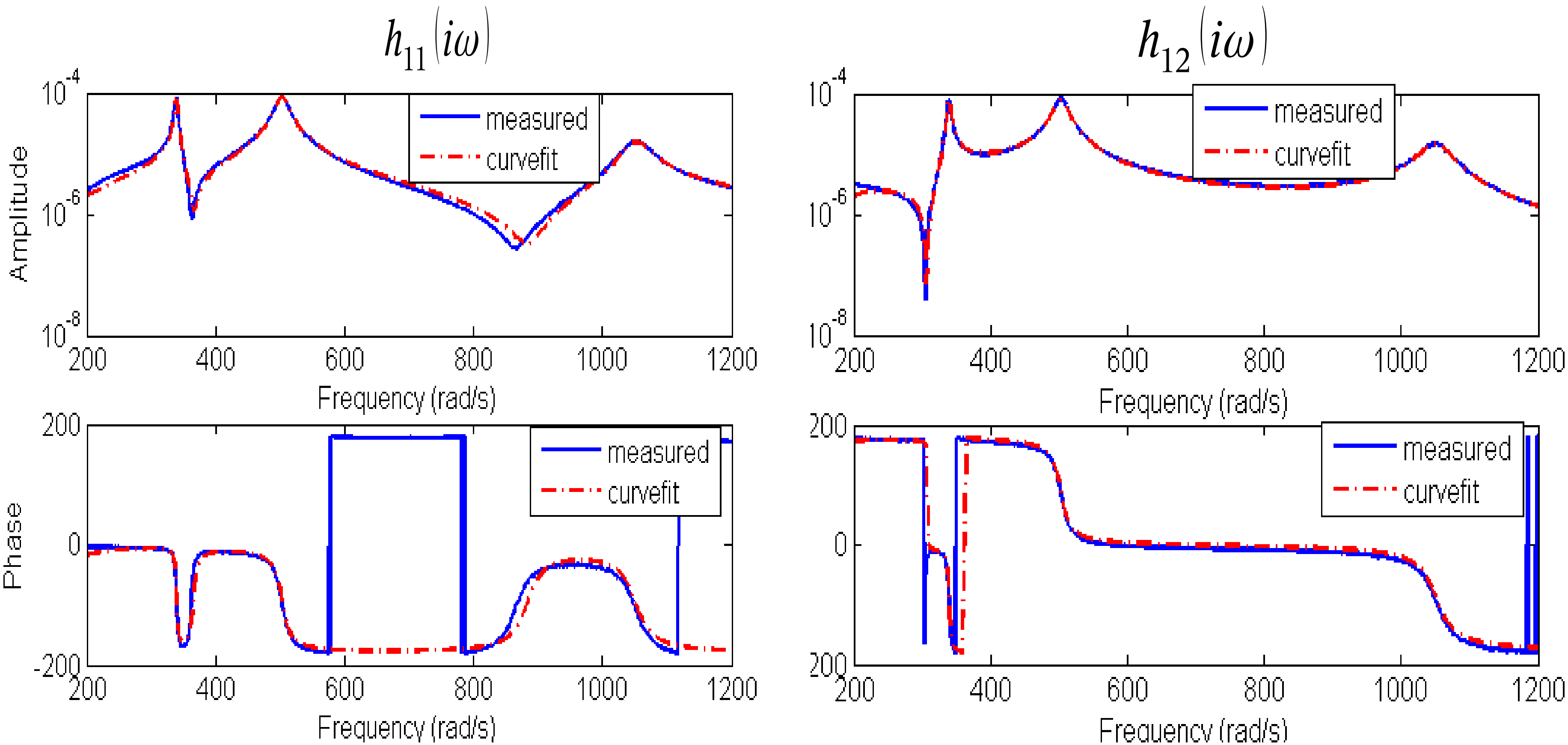




(a) ‘T’ configuration

(b) ‘H’ configuration

T-Configuration

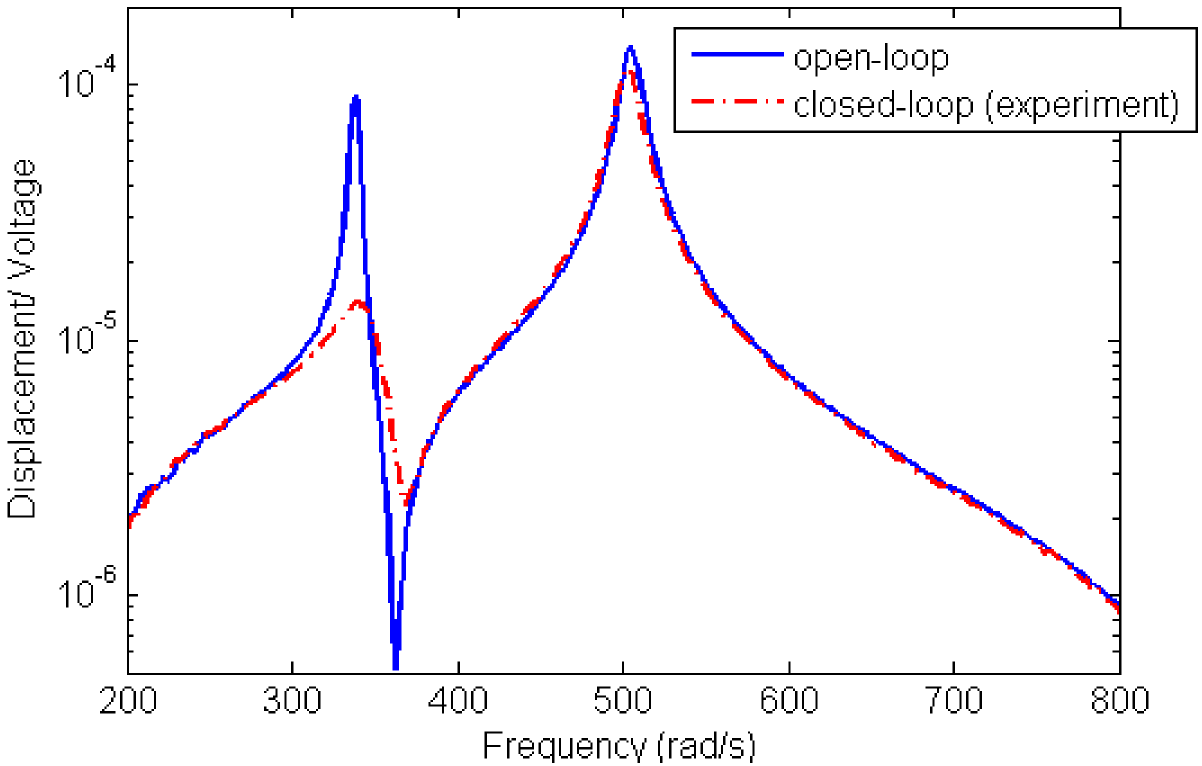


Open-loop poles:

$$\lambda_{1,2} = -2.0 \pm 339i$$
$$\lambda_{3,4} = -8.7 \pm 503i$$
$$\lambda_{5,6} = -20.0 \pm 1050i$$

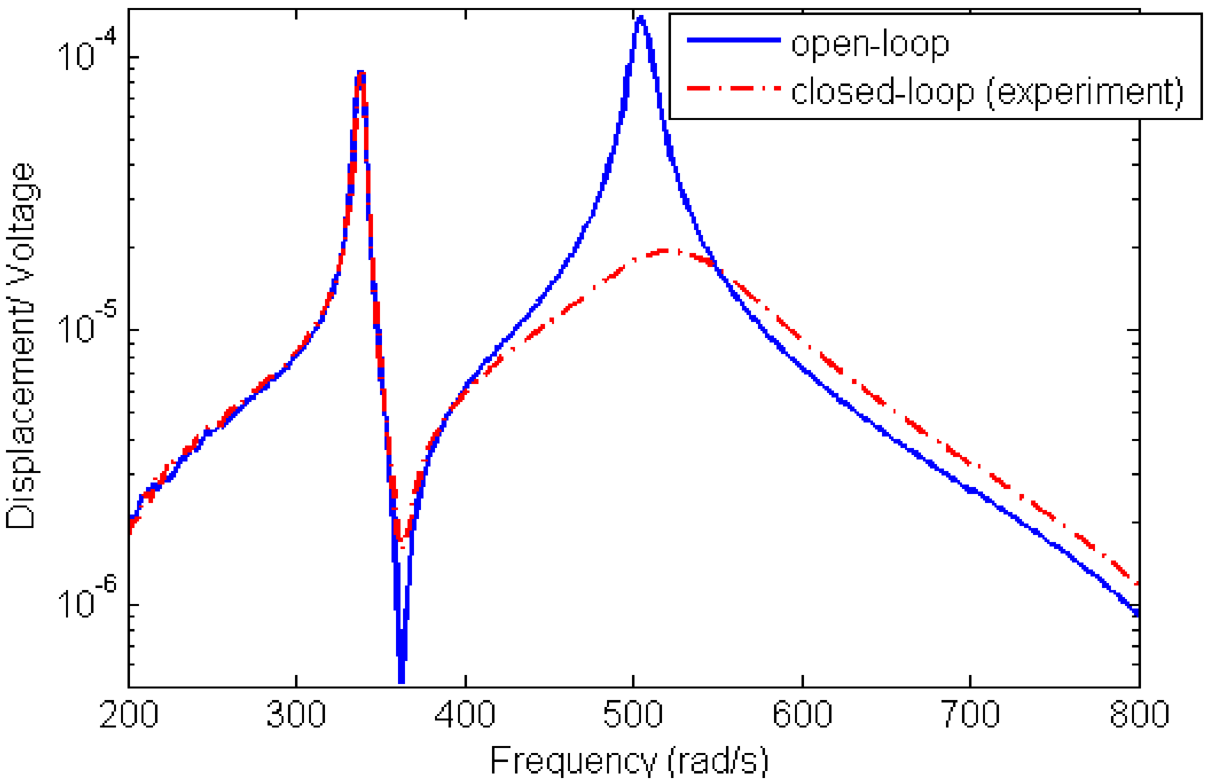
Assignment of the Bending Mode

$$b=\begin{pmatrix}1\\1\end{pmatrix} \qquad \mu_{1,2}=-8\pm 350\,i \qquad g=\begin{bmatrix}19000\\19000\end{bmatrix} \qquad f=\begin{bmatrix}34\\34\end{bmatrix}$$

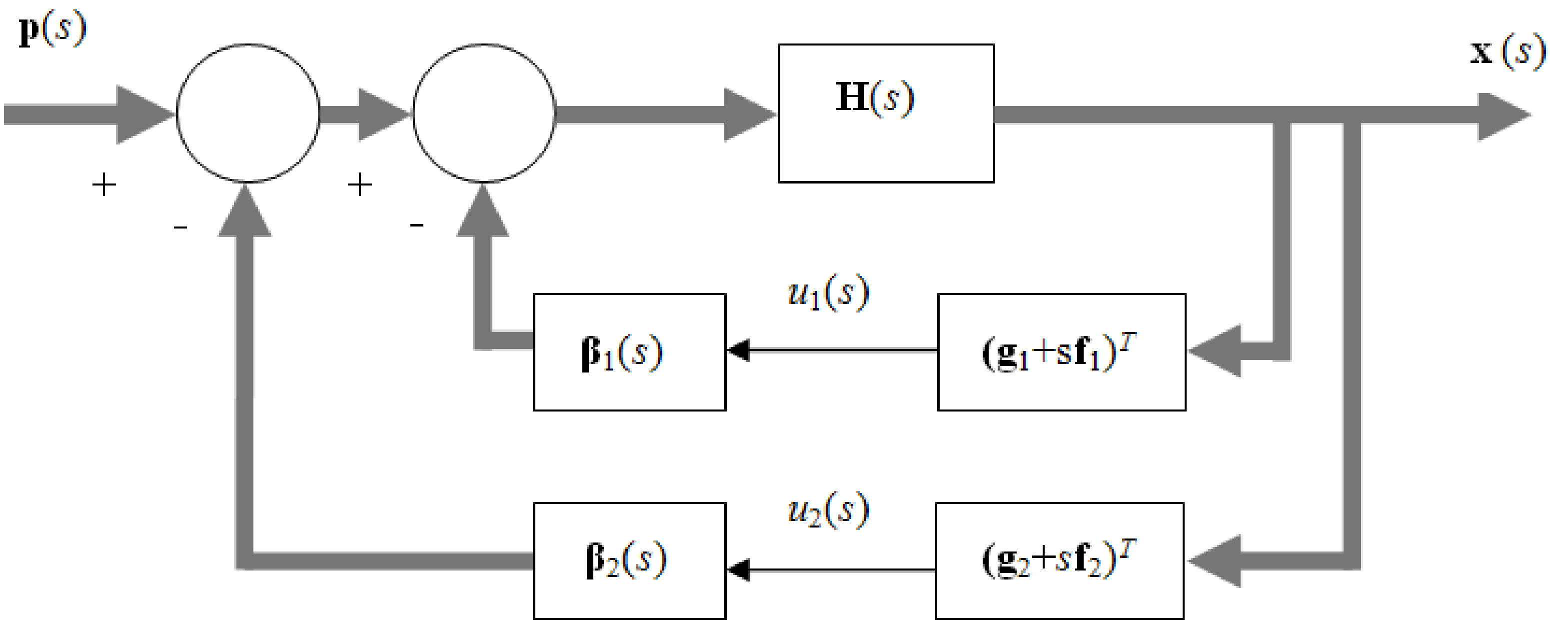


Assignment of the Torsional Mode

$$b=\begin{pmatrix}1\\-1\end{pmatrix} \qquad \mu_{1,2}=-60\pm 535\,i \qquad g=\begin{bmatrix}10862\\-10862\end{bmatrix} \quad f=\begin{bmatrix}30\\-30\end{bmatrix}$$



Sequential Pole Placement using Multi-input State Feedback

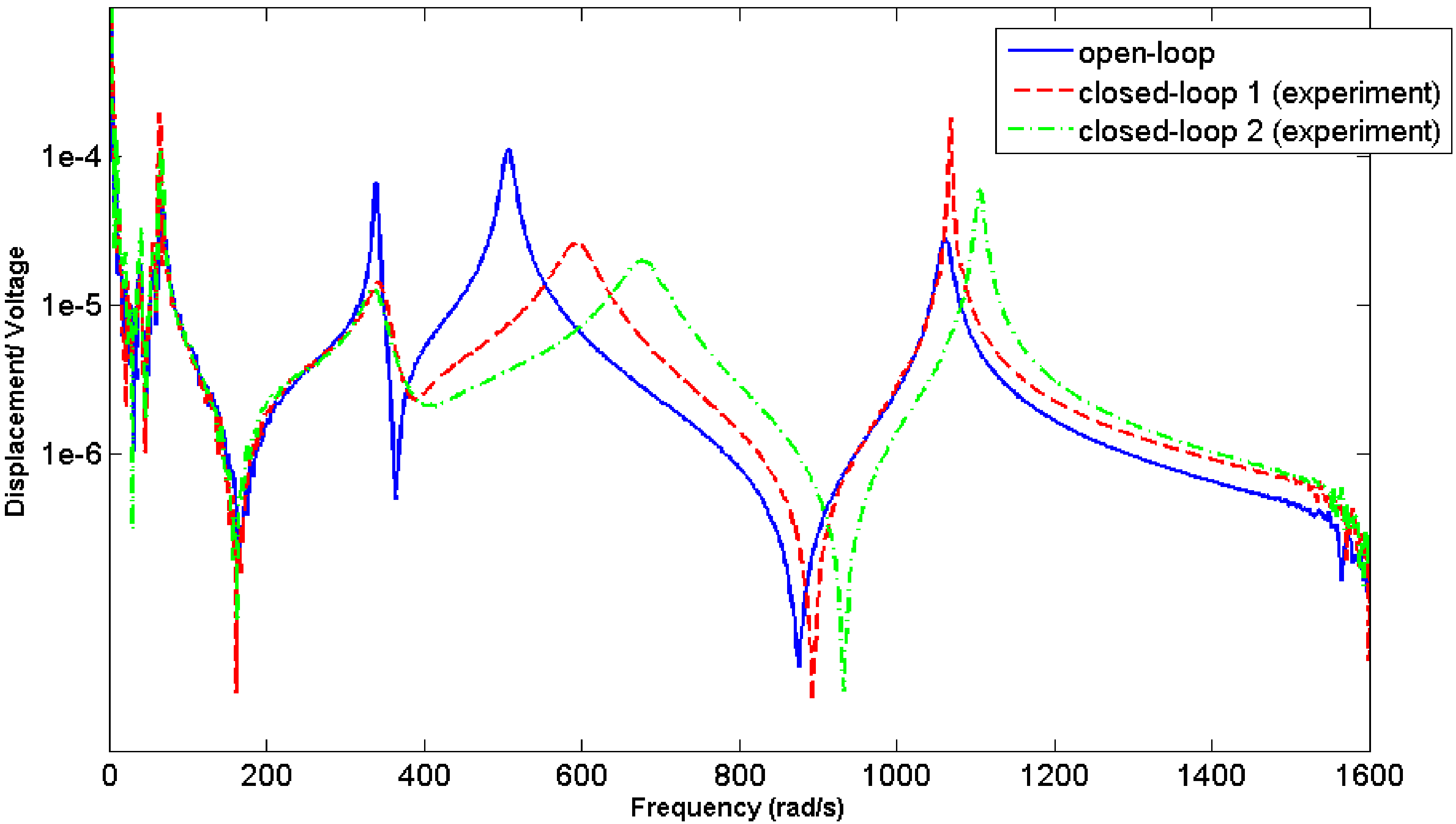


$$\det\left(I+\left(G+\mu_j F\right)^T H\left(\mu_j\right) B\left(\mu_j\right)\right)=0$$

$$B(s)=\left[\beta_1(s) \quad \beta_2(s) \quad \cdots \quad \beta_n(s)\right];$$
$$G=\left[g_1 \quad g_2 \quad \cdots \quad g_n\right]; \; F=\left[f_1 \quad f_2 \quad \cdots \quad f_n\right]$$

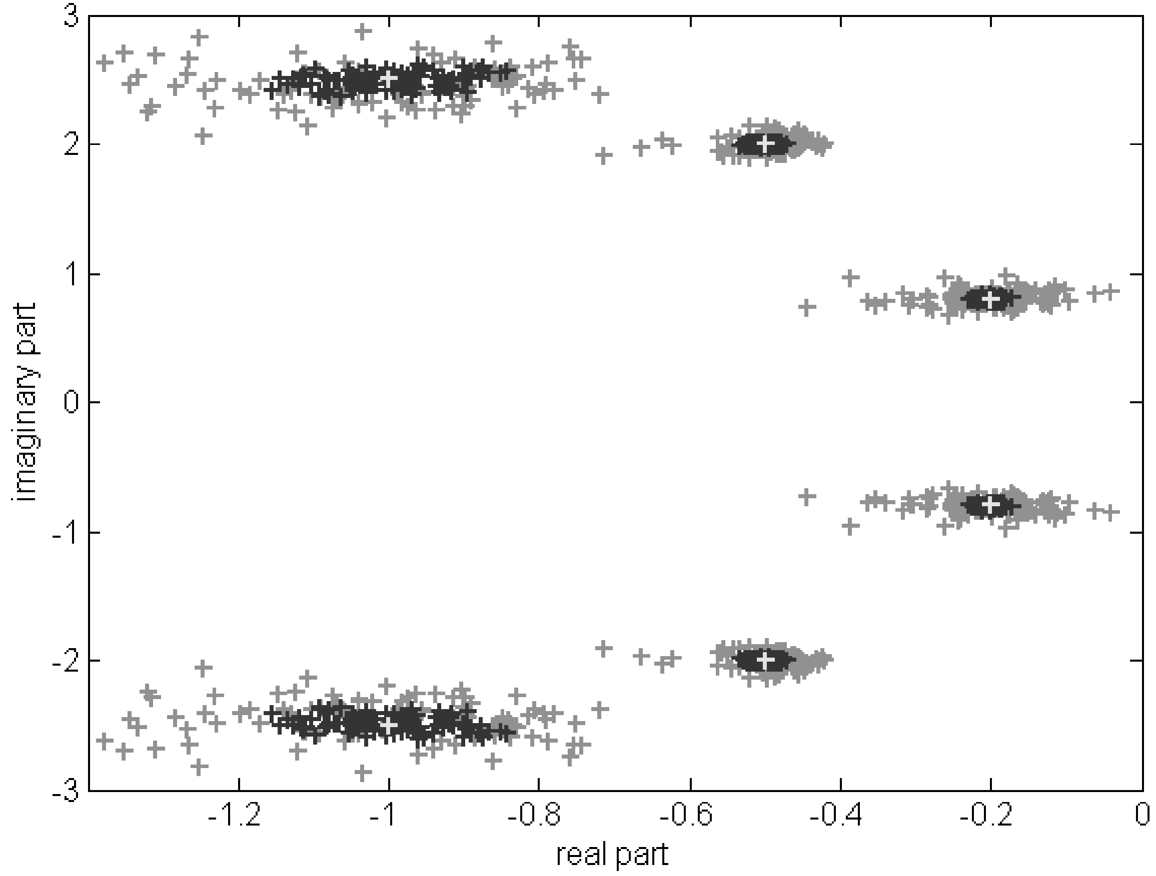
Multi-input State Feedback

$$\mu_{1,2} = -13 \pm 350\,i \qquad B = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad G = \begin{bmatrix} 17900 & 34000 \\ 17900 & -34000 \end{bmatrix} \qquad F = \begin{bmatrix} 61 & 13 \\ 61 & -13 \end{bmatrix}$$
$$\mu_{3,4} = -30 \pm 610\,i$$



Robust Pole Placement

Assignment of poles by single-input and multi-input control



Robustness to Measurement Errors

The poles are given by the zeros of $p(s)$, $s=\mu_j$

$$p\left(H,\ \mu_j\right)=1+\left(g+\mu_j f\right)^T H\left(\mu_j\right)\ b=0$$

Consider a small perturbation $H+\sum_{p=1}^{n}\sum_{q=1}^{n}\delta h_{pq}\, e_p e_q^T$ resulting in $\mu_j+\delta\mu_j$

$$p\left(H+\sum_{p=1}^{n}\sum_{q=1}^{n}\delta h_{pq}\, e_p e_q^T,\ \mu_k\right)=p\left(H,\ \mu_k\right)+\sum_{p=1}^{n}\sum_{q=1}^{n}\left(\frac{\partial p}{\partial h_{pq}}\right)\delta h_{pq}=0$$

which leads to,

$$\frac{\partial \mu_k}{\partial h_{pq}}=\frac{-\left(g+\mu_k f\right)^T e_p e_q^T b\left(\mu_k\right)}{f^T H\left(\mu_k\right) b\left(\mu_k\right)+\left(g+\mu_k f\right)^T\left(\frac{\partial H}{\partial s}\Big|_{s=\mu_k} b\left(\mu_k\right)+H\left(\mu_k\right)\frac{\partial b}{\partial s}\Big|_{s=\mu_k}\right)-\left(g+\mu_k f\right)^T e_p \frac{\partial h_{pq}}{\partial s}\Big|_{s=\mu_k} e_q^T b\left(\mu_k\right)}$$

Example: Robust Pole Placement

$$M=\begin{bmatrix}2 & & \\ & 1 & \\ & & 3\end{bmatrix}\quad C=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & -1\\ & -1 & 1\end{bmatrix}\quad K=\begin{bmatrix}6 & -2 & -1\\ -2 & 4 & -2\\ -1 & -2 & 3\end{bmatrix}$$

We wish to assign the closed-loop poles while using the robustness condition,

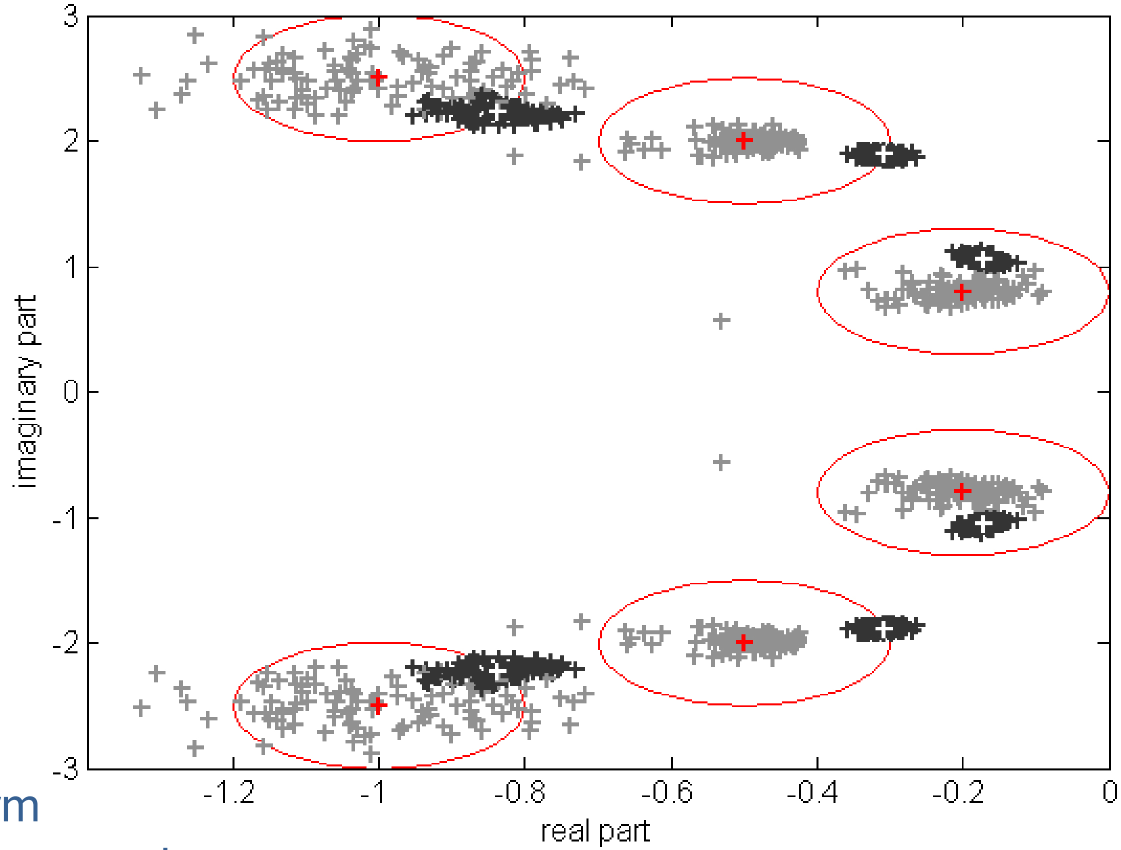
$$\min_{g,\,f}\left\| \frac{\partial \mu_k}{\partial h_{11}} \quad \frac{\partial \mu_k}{\partial h_{12}} \quad \cdots \quad \frac{\partial \mu_k}{\partial h_{3,3}} \right\|;\quad k=1,\,2,\,\ldots,\,6$$

Single-input robust assignment of poles

The poles are assigned to elliptical regions with centres:

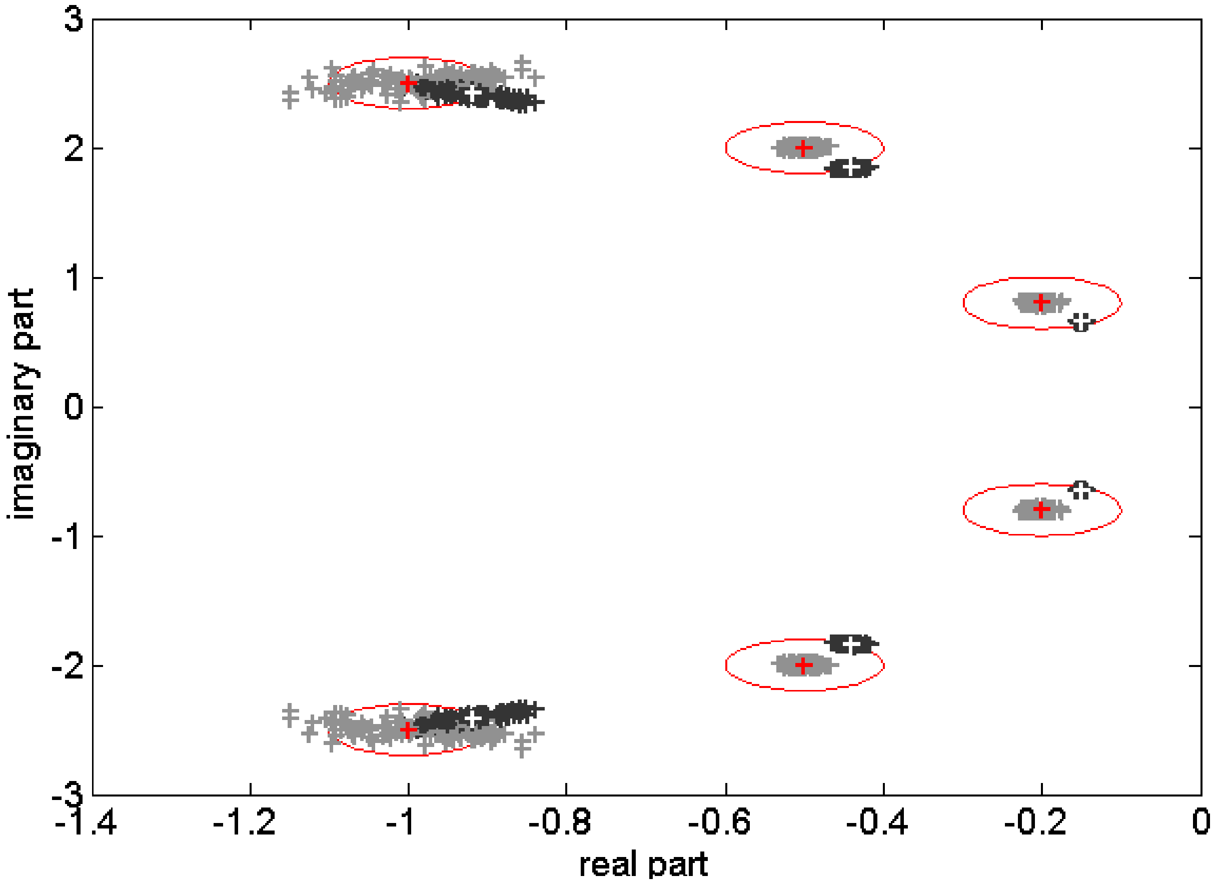
$$\left(x_{c1,2}, y_{c1,2}\right)=\left(-0.2,\pm 0.8\right)$$
$$\left(x_{c3,4}, y_{c3,4}\right)=\left(-0.5,\pm 2\right)$$
$$\left(x_{c5,6}, y_{c5,6}\right)=\left(-1,\pm 2.5\right)$$

and semi-axes a=0.2 and b=0.5.



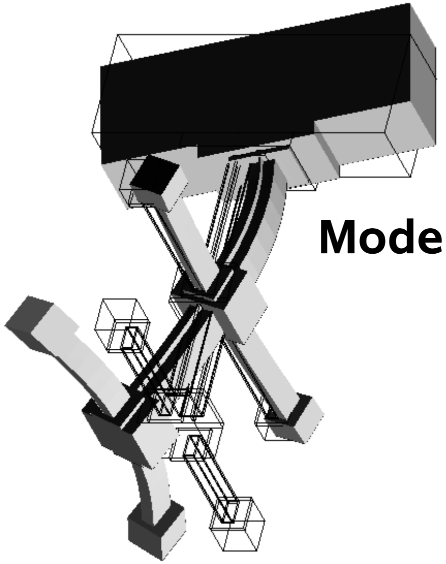
100 samples taken from a uniform distribution representing a measurement error of ± 10 on $H\left(\mu_j\right)$

Sequential multi-input robust assignment of poles

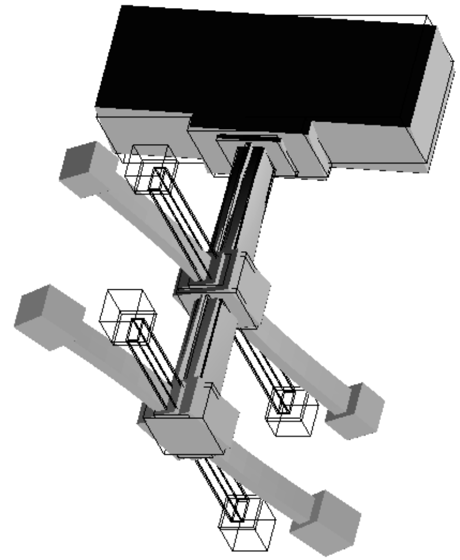


M.G. Tehrani, J.E. Mottershead, A.T. Shenton and Y. M. Ram, Robust pole placement in structures by the method of receptances, Mechanical Systems and Signal Processing, 2011, 25(1),112-122.

H-Configuration



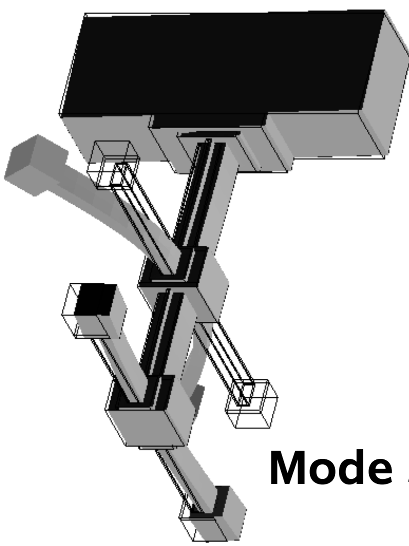
Mode 1: 54 Hz



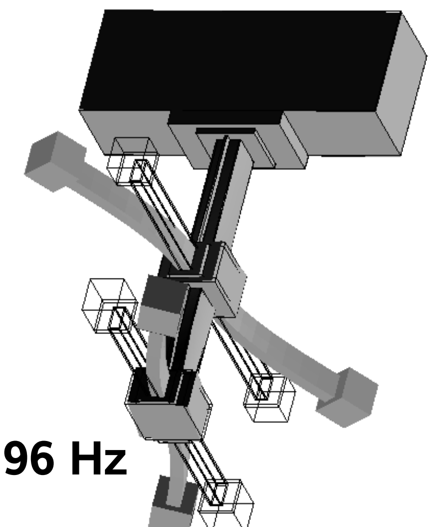
Mode 2:
60Hz

Bending modes

Torsional modes



Mode 3: 86 Hz



Mode 4: 96 Hz

Partial Pole Placement of the Bending Modes (1&3)

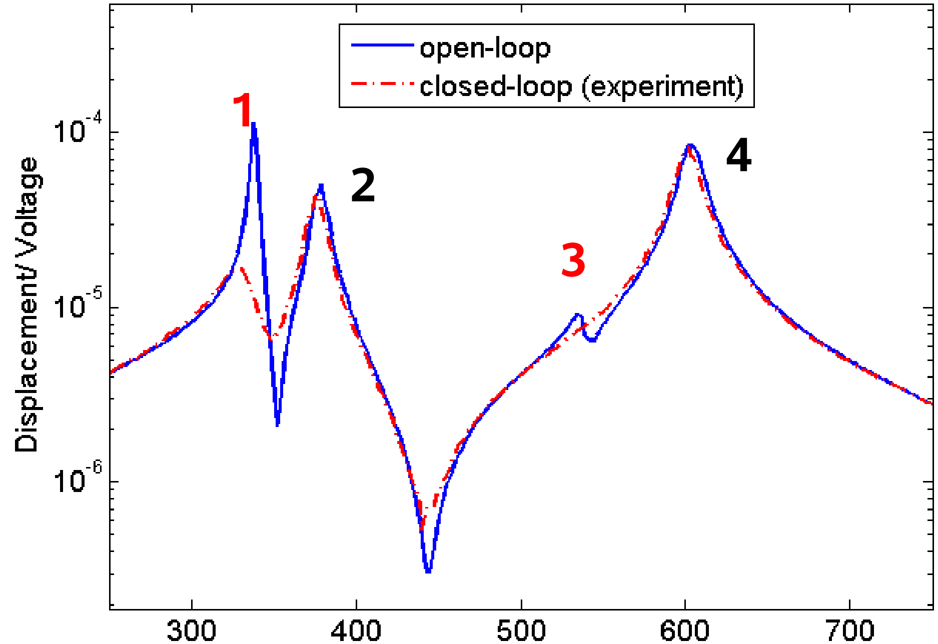
$$\mu_{1,2} = -12 \pm 340i$$

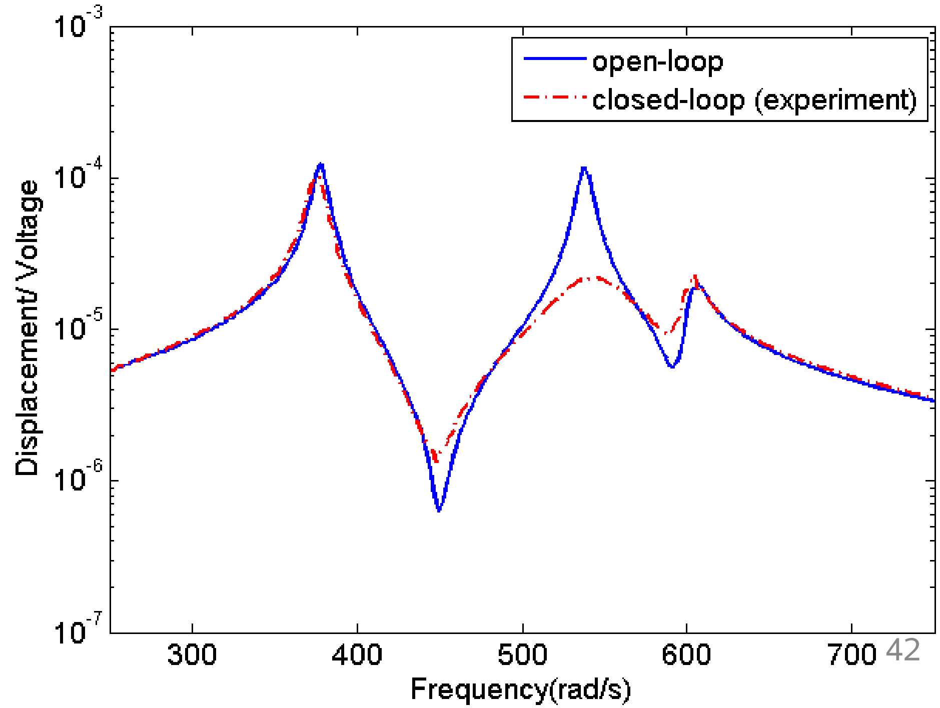
$$\mu_{5,6} = -30 \pm 550i$$

$$b = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T$$

$$f = \begin{bmatrix} 57 & 57 & 32 & 32 \end{bmatrix}^T$$

$$g = \begin{bmatrix} 5700 & 5700 & 9760 & 9760 \end{bmatrix}^T$$





Partial Pole Placement of the Torsional Modes (2&4)

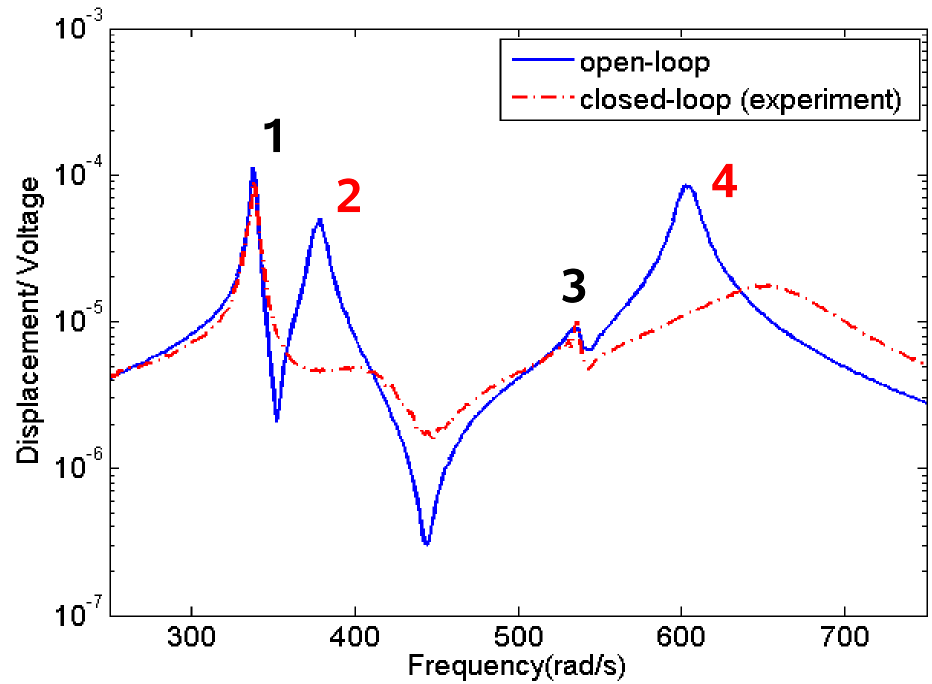
$$\mu_{3,4} = -12 \pm 410\,i$$

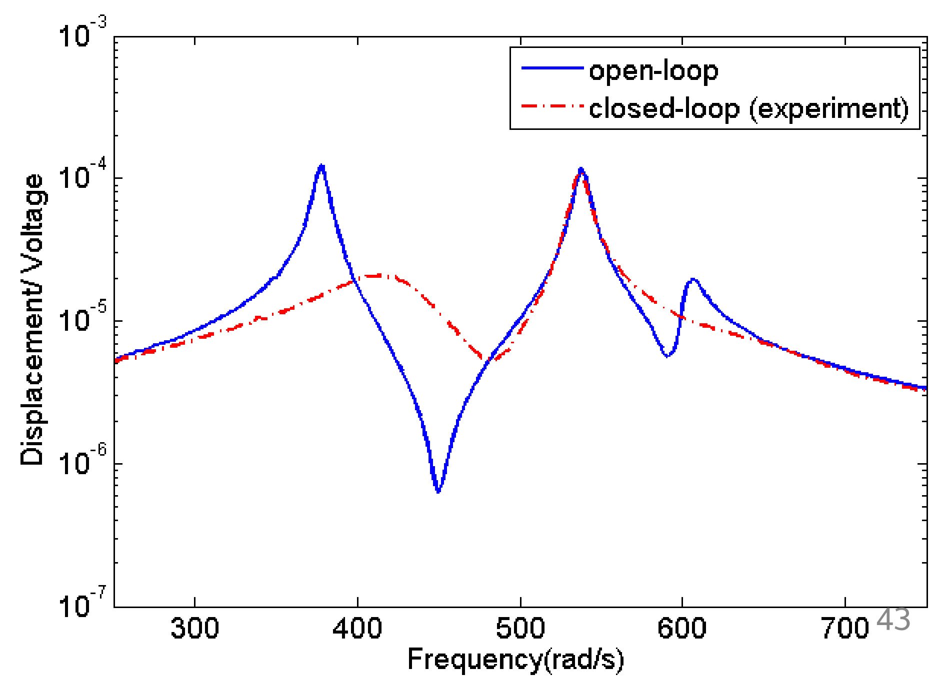
$$\mu_{7,8} = -65 \pm 640\,i$$

$$b = \begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix}^T$$

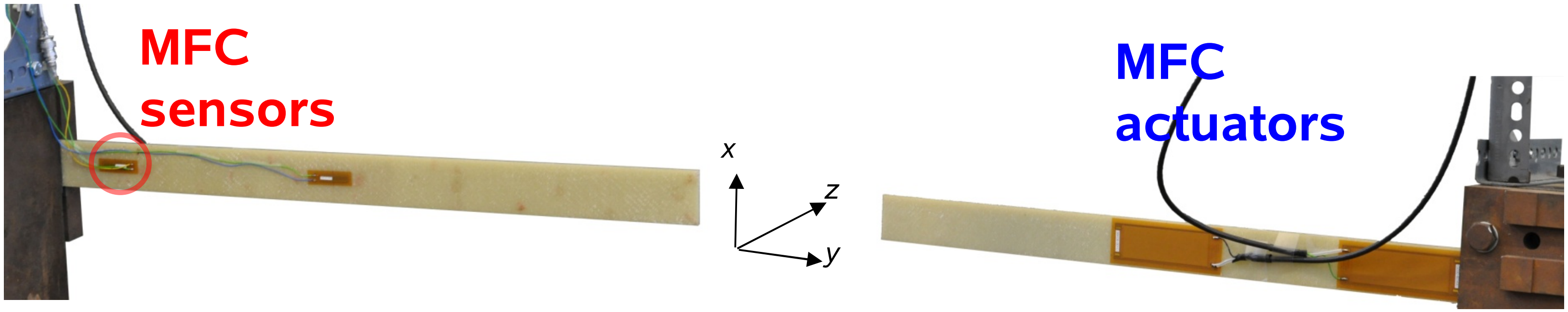
$$f = \begin{bmatrix} -55 & 55 & 16 & -16 \end{bmatrix}^T$$

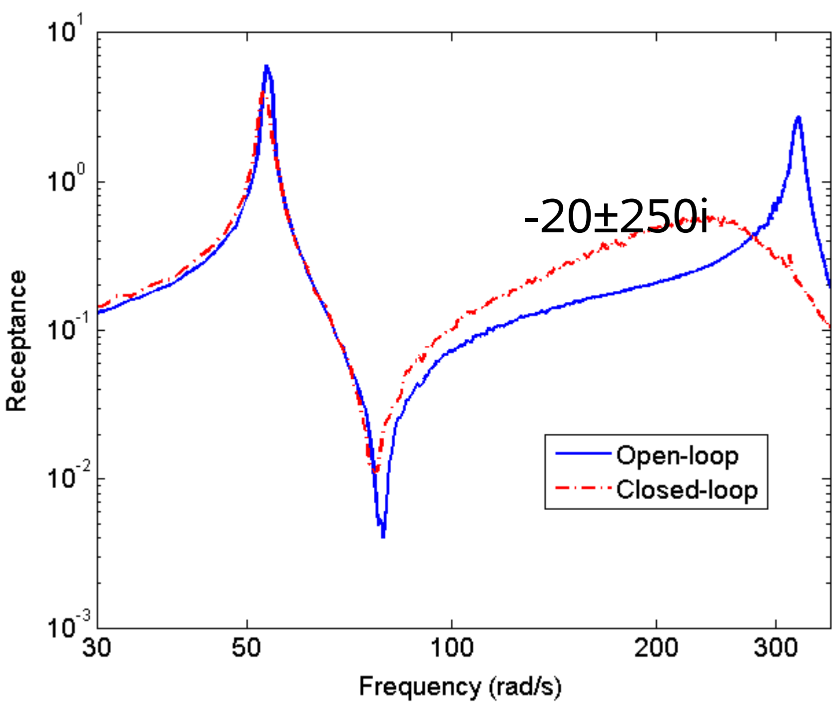
$$g = \begin{bmatrix} 20000 & -20000 & 840 & -840 \end{bmatrix}^T$$

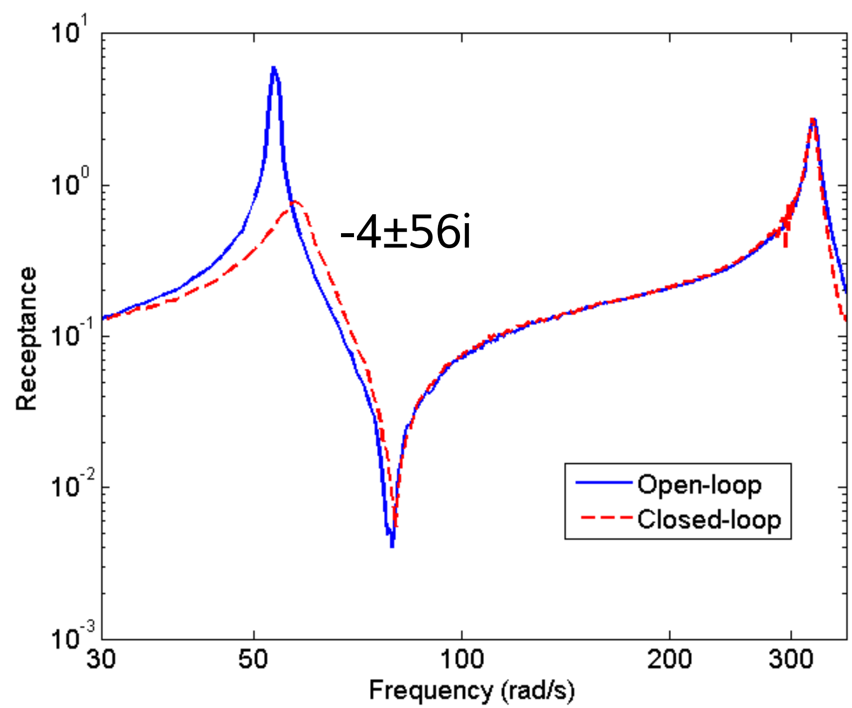




Piezo Beam





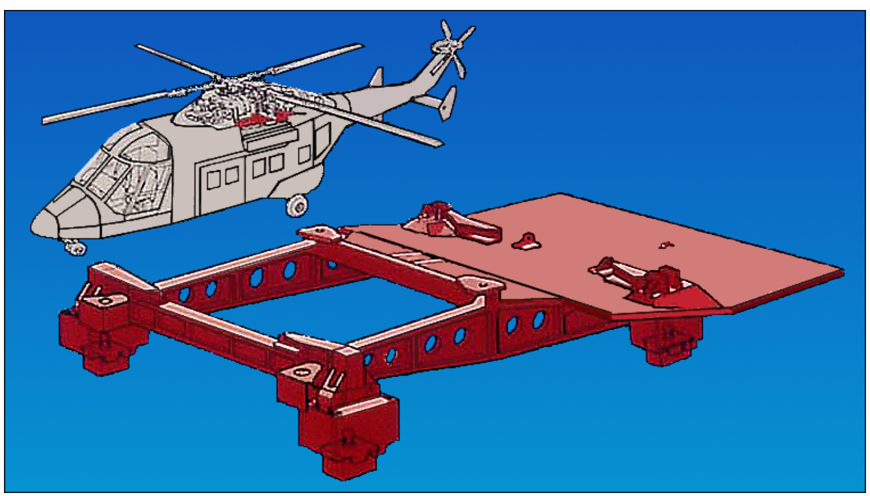


M.G. Tehrani, R. N. R. Elliott and J. E. Mottershead, Partial Pole Placement in Structures by the Method of Receptances: Theory and Experiments, *Journal of Sound and Vibration*, 2010, 329(24) 5017–5035.

AgustaWestland W30 Helicopter Airframe

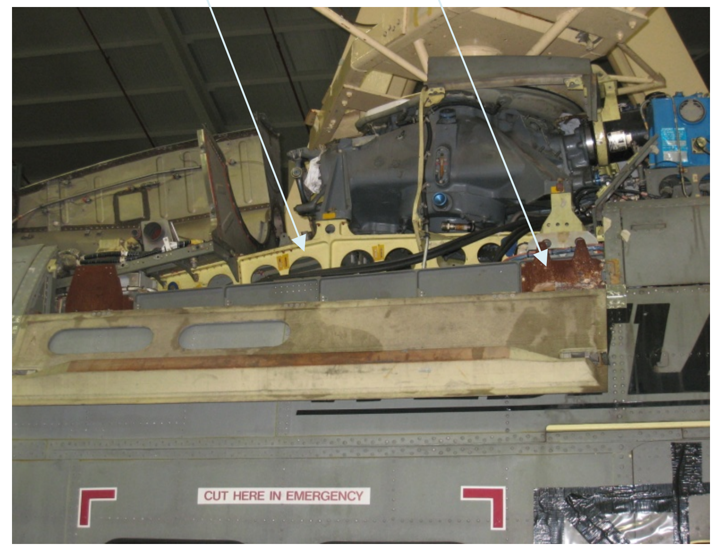
- Experiments carried out on AgustaWestland W30 helicopter airframe at Yeovil
- In total five days (two visits to Yeovil) during February and March 2011
- We used electro-hydraulic actuators built in the airframe for excitation and control
- Experiments include:
  - Open-loop tests with two different input voltages
  - Closed-loop tests with the higher input voltage
- The airframe system is nonlinear
- The closed-loop poles were assigned with small real parts so that the sharp peaks would be clearly seen in the measured closed-loop FRF
- Motivation: to avoid the resonance due to the blade passing frequency

W30 Helicopter Airframe

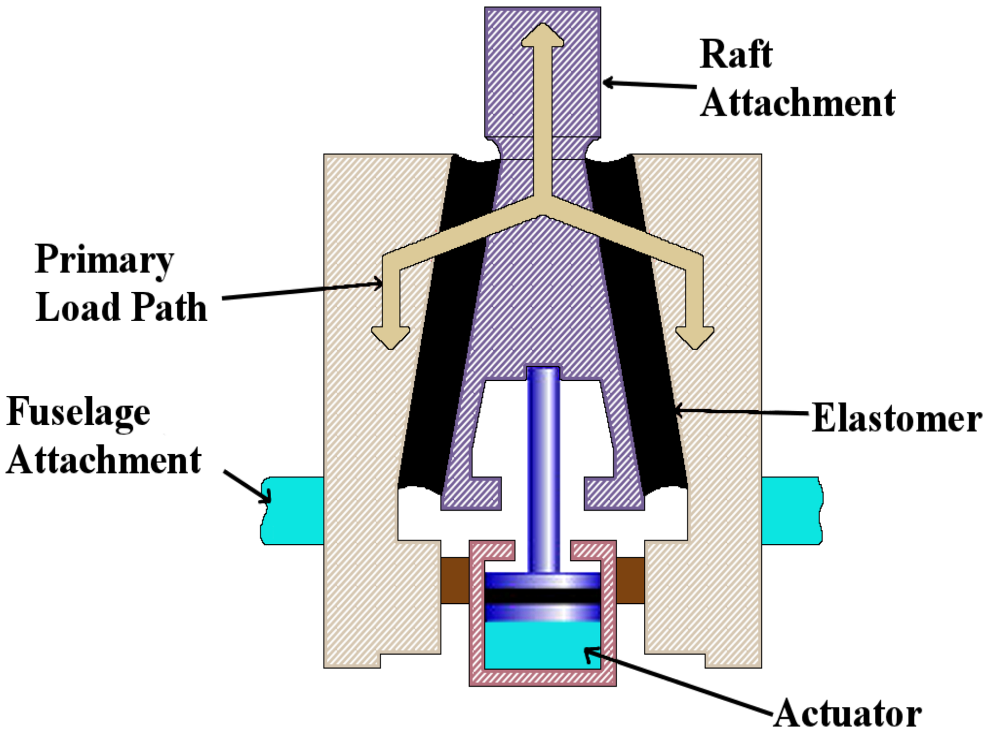


Raft

Actuator



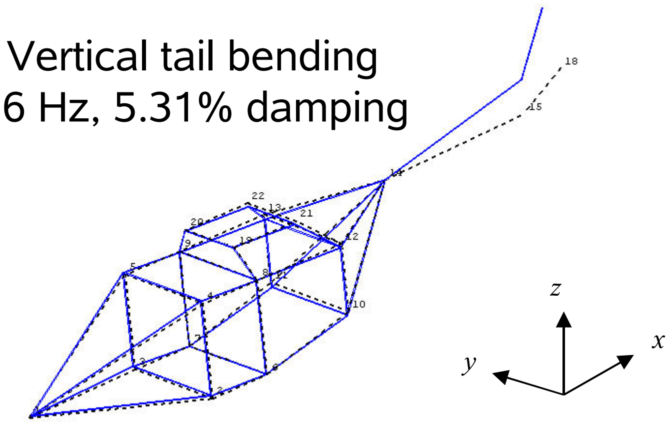
Electro-hydraulic Actuators



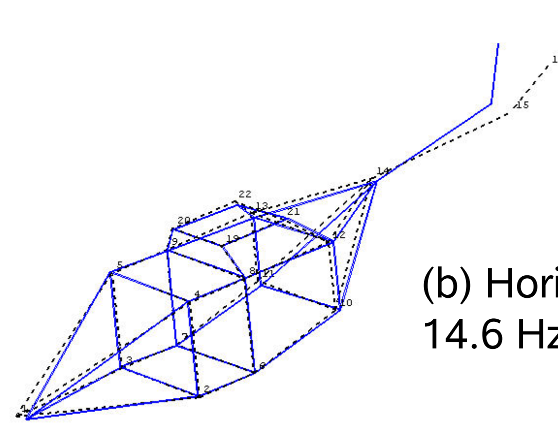
- There are four actuators in total.
- The main gearbox was present but the engines had been removed.
- Insufficient mass over the rear actuators for reacting the control force.
- The 2 rear actuators were pressurised and used passively.
- The 2 front actuators were used to apply the control force.

Open-Loop Modes

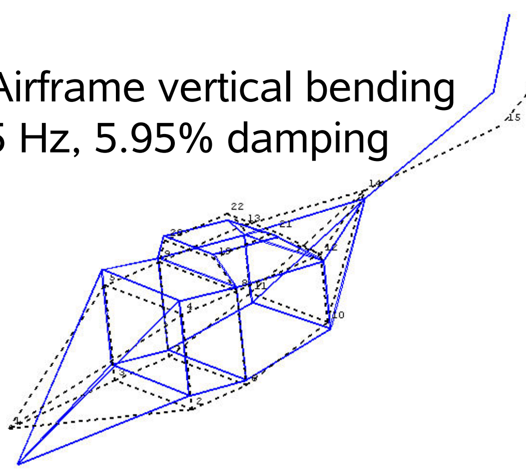
(a) Vertical tail bending
12.6 Hz, 5.31% damping



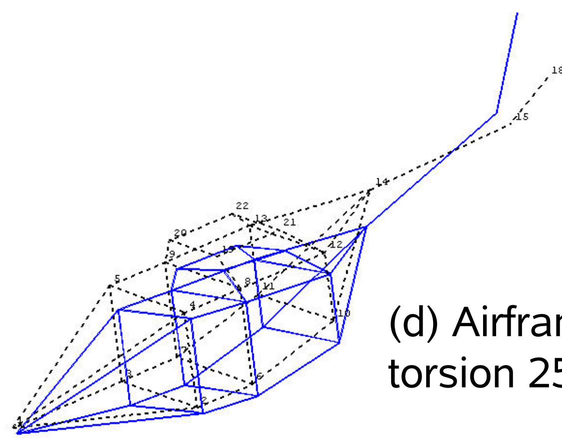
(b) Horizontal tail bending
14.6 Hz, 2.78% damping



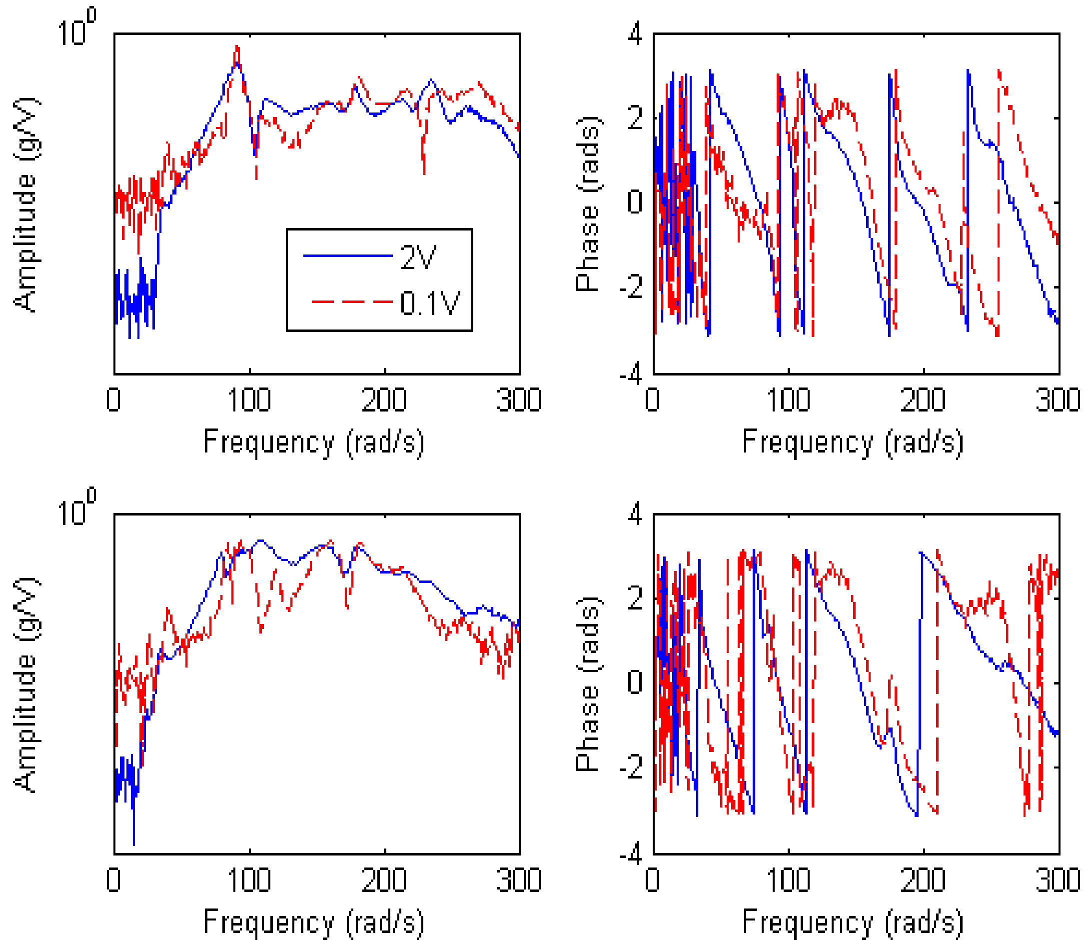
(c) Airframe vertical bending
17.5 Hz, 5.95% damping



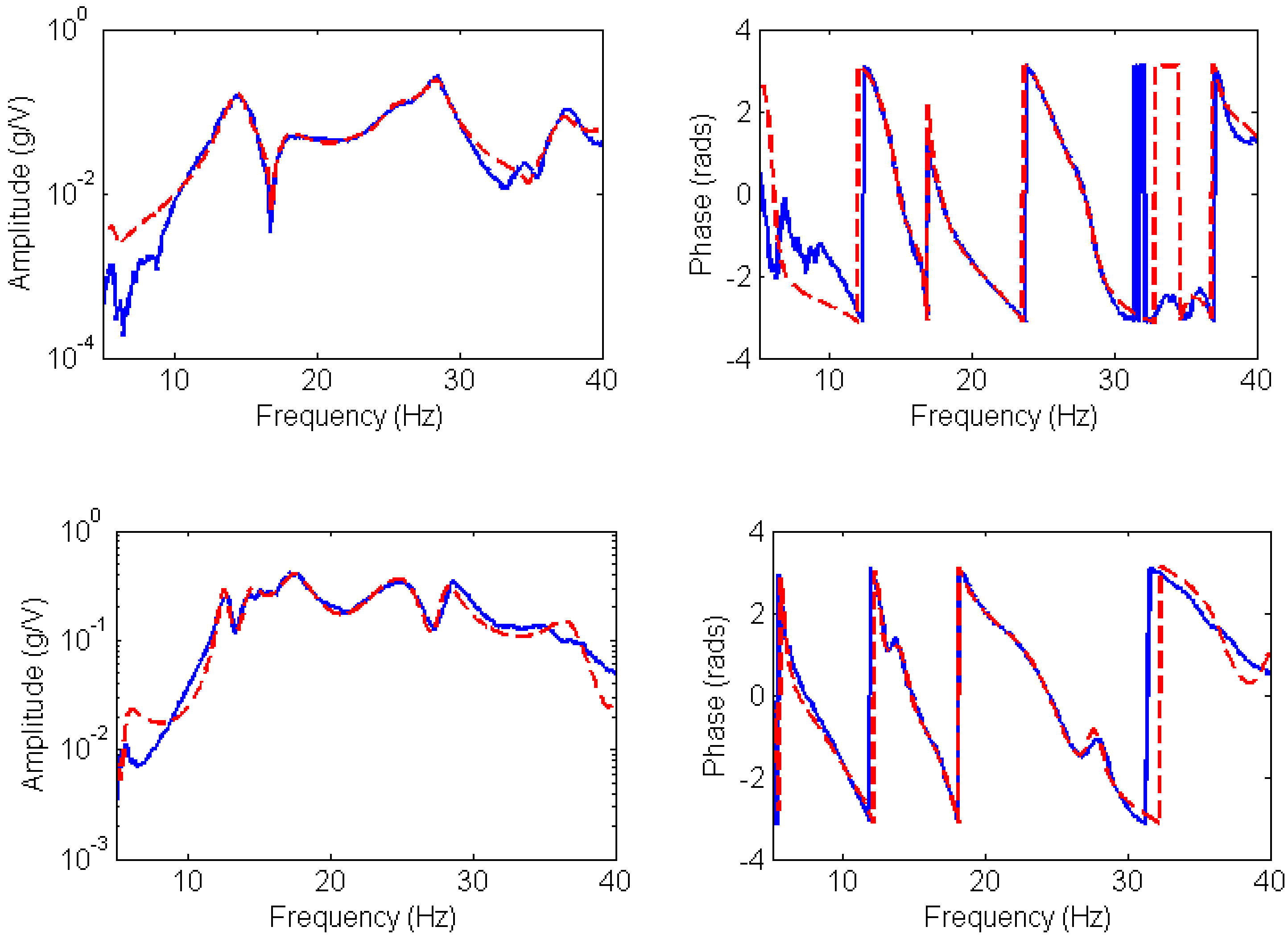
(d) Airframe horizontal bending and torsion 25.4 Hz, 5.93% damping



Nonlinearity: FRFs at Different Amplitudes

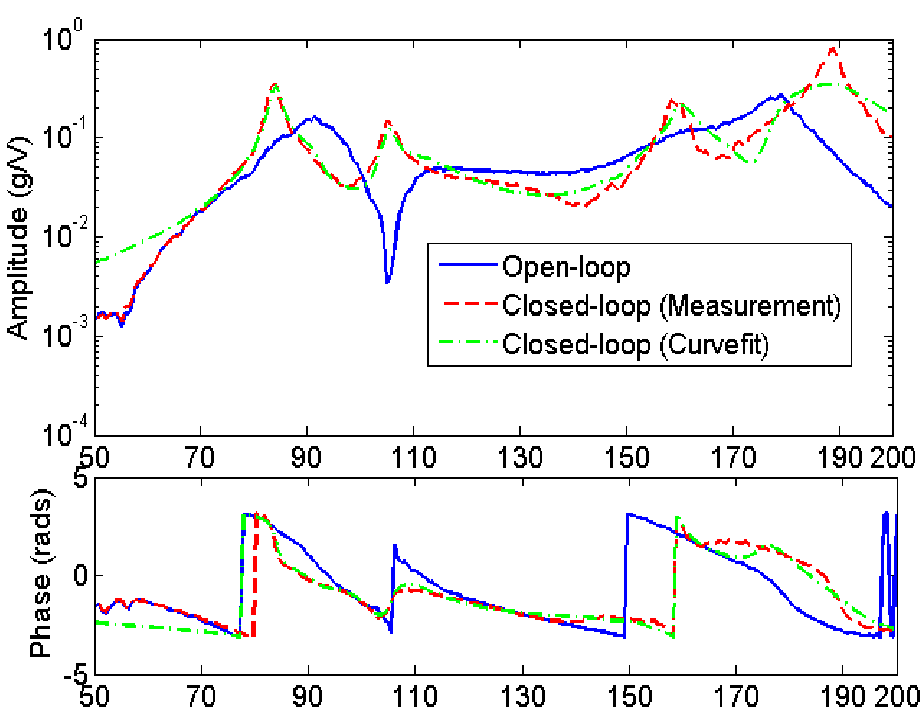


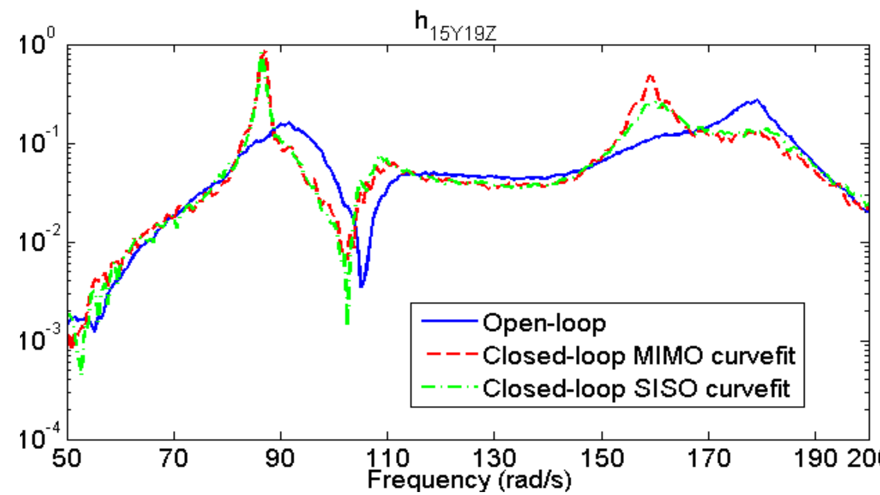
Example Measured and Curve-Fitted FRFs



Pole Placement: Simulated and Experimental Results

$$b = \begin{pmatrix} 1 & 1 \end{pmatrix}^T \qquad \mu_{1,2} = -1 \pm 84i,\ \mu_{3,4} = -2 \pm 160i$$

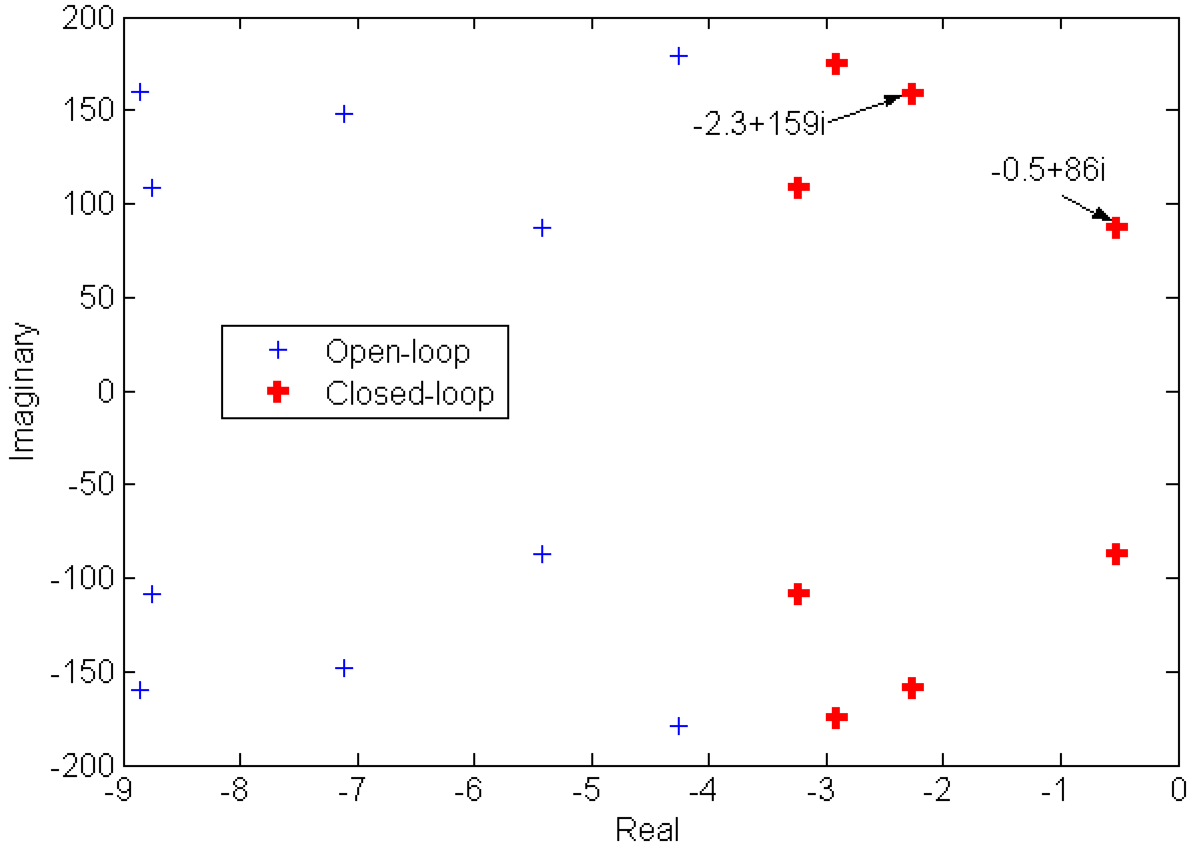




Experiment

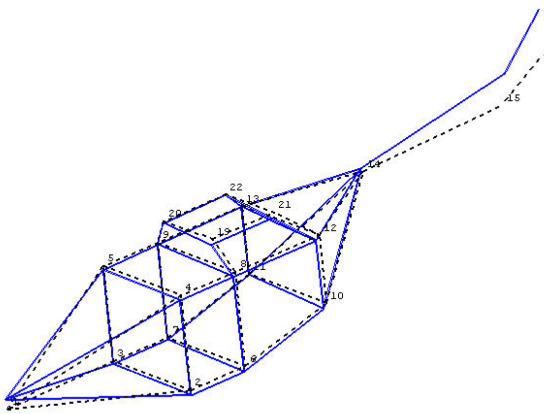
Simulation

Location of the closed-loop poles (1st case)



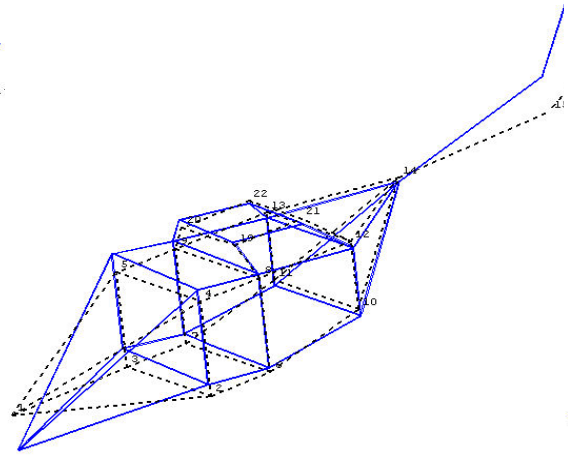
Pole *s*-plane locations

Closed-loop mode-shapes (1st case)



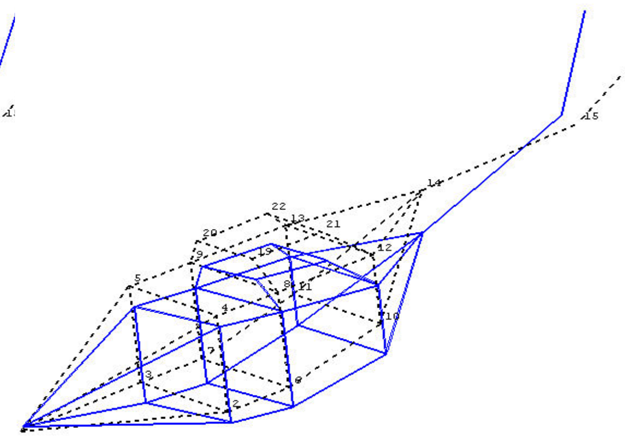
(a) Tail bending:
vertical + horizontal

$13.7\,\mathrm{Hz}\ (86\ \mathrm{rad/s})$
0.98 damping



(b) Vertical bending
of airframe

$17.0\,\mathrm{Hz}\ (107\ \mathrm{rad/s})$
2.18 damping



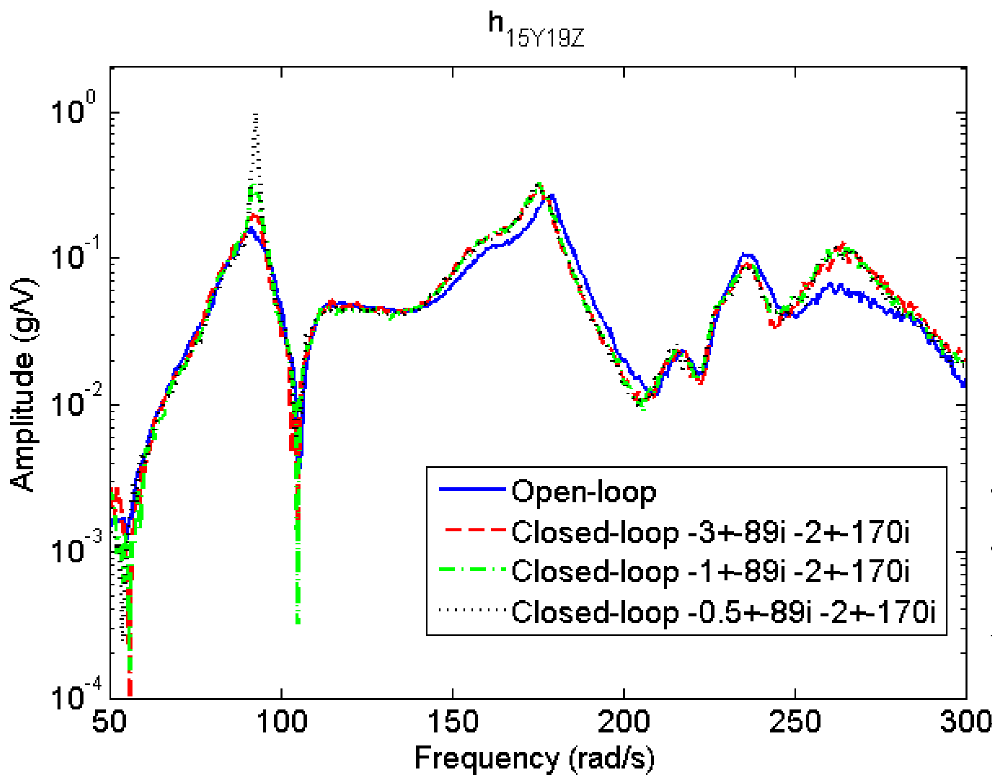
(c) Horizontal bending
of airframe

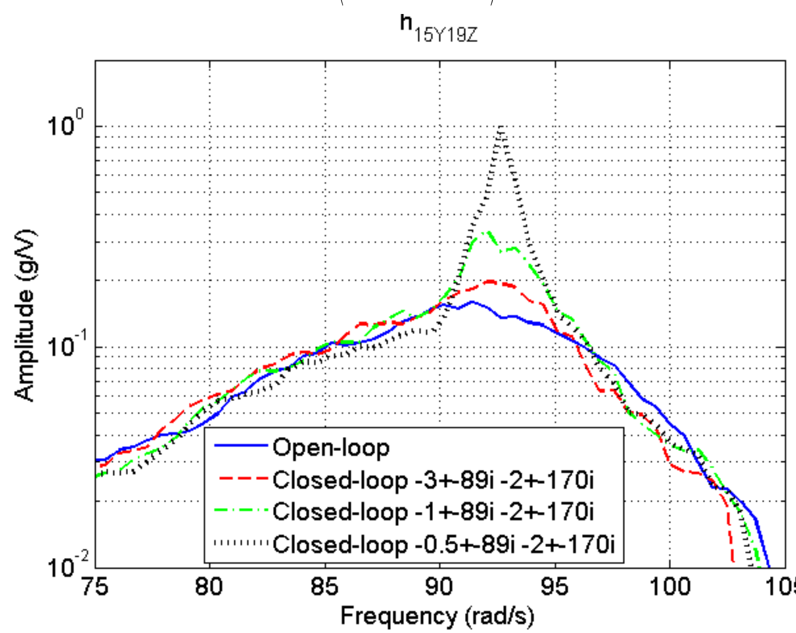
$25.2\,\mathrm{Hz}\ (157\ \mathrm{rad/s})$
1.28 damping

Other Experimental Results: Assigning 3 Different Levels of Damping

$$\mu_{1,2} = -3 \pm 89i\,,\ \mu = -8 \pm 170i$$
$$\mu_{1,2} = -1 \pm 89i\,,\ \mu = -2 \pm 170i$$
$$\mu_{1,2} = -0.5 \pm 89i\,,\ \mu = -2 \pm 170i$$

$$b = \begin{pmatrix} 1 & -1 \end{pmatrix}^T$$





J.E. Mottershead and **M.G. Tehrani,** S. James and P. Court, Active Vibration Control Experiments on an Agusta-Westland W30 Helicopter Airframe, *Journal of IMECHE part C*, 2011, in press.

Future Work

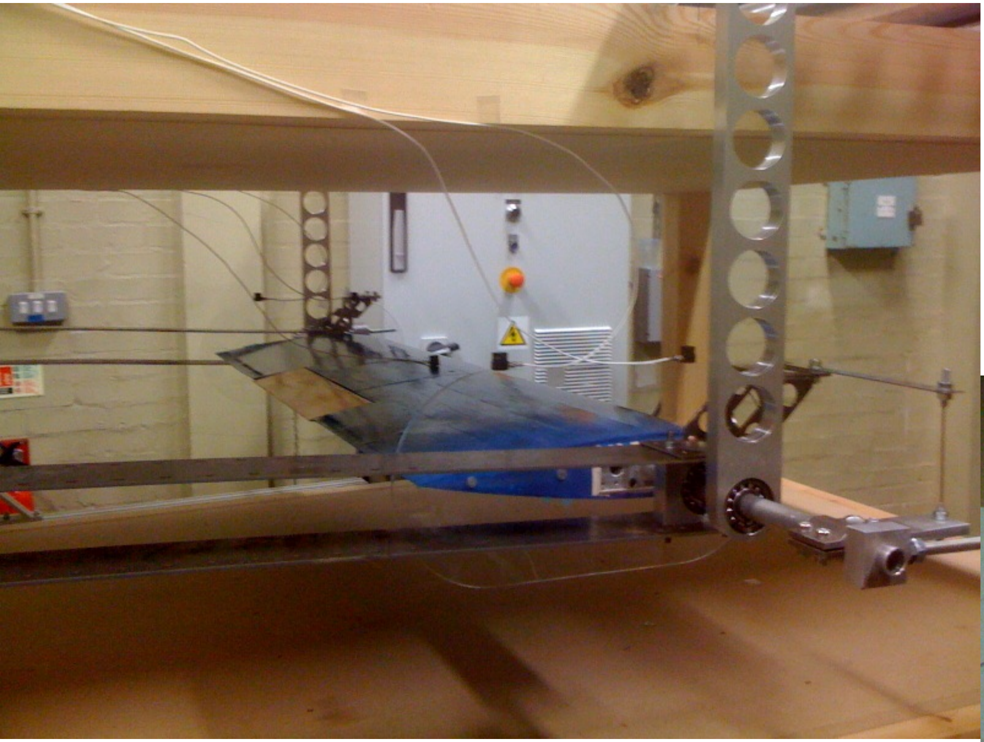
- Active flutter control for aircraft.
- Active control of asymmetric systems.
- Stochastic control.
- Vibration control of parametrically excited systems.

Future work: Aeroservoelasticity

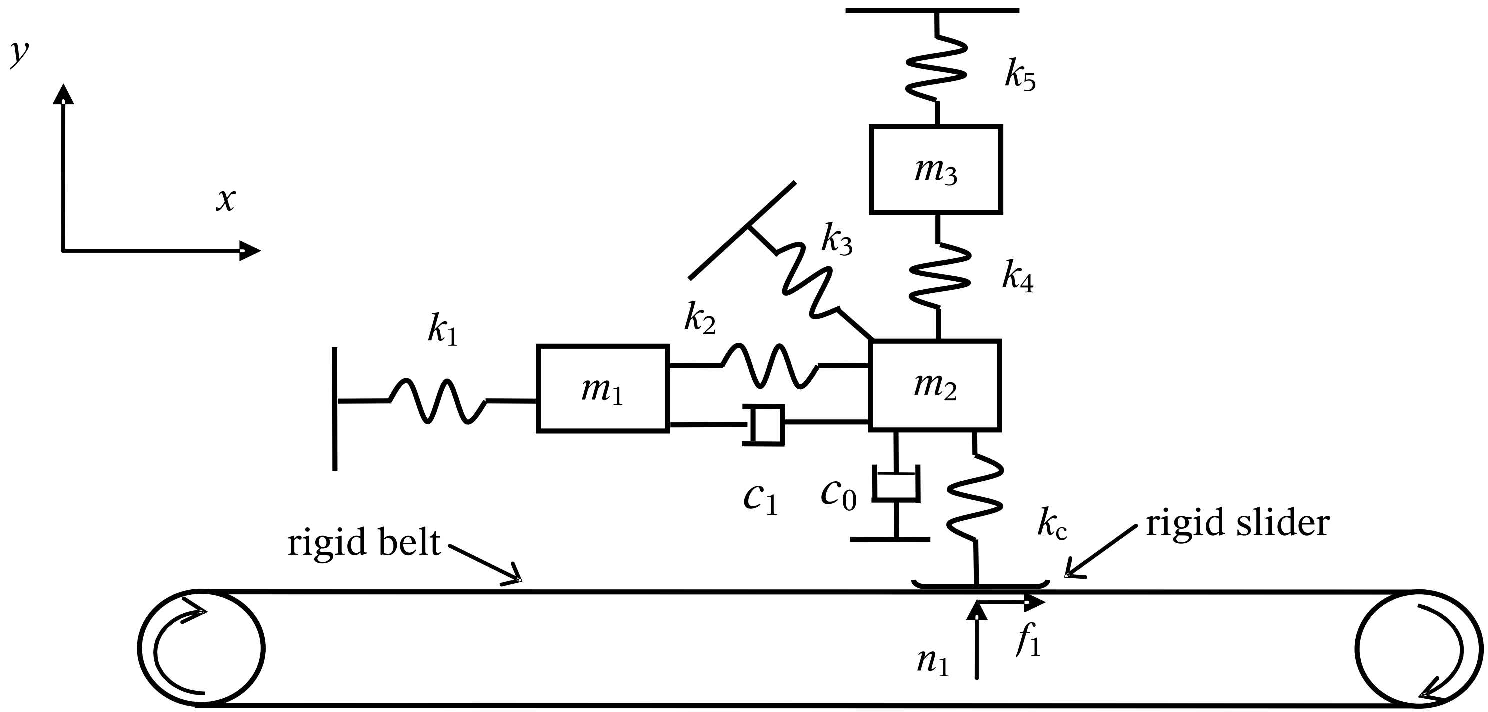
Motivation: To increase the flutter boundary by the eigenvalue assignment

Dynamics and Control Laboratory at the University of Liverpool

Aeroservoelasticity



Future work: Asymmetric Systems



$$M = \begin{bmatrix} m_1 & & & \\ & m_3 & & \\ & & m_2 & \\ & & & m_2 \end{bmatrix} \quad C = \begin{bmatrix} c_1 & 0 & -c_1 & 0 \\ 0 & 0 & 0 & 0 \\ -c_1 & 0 & c_1 & 0 \\ 0 & 0 & 0 & c_0 \end{bmatrix} \quad K = \begin{bmatrix} k_1+k_2 & 0 & -k_2 & 0 \\ 0 & k_4+k_5 & 0 & 0 \\ -k_2 & 0 & k_2+0.5\mathrm{k}_3 & 0.5\mathrm{k}_3 \\ 0 & 0 & 0.5\mathrm{k}_3 & k_4+0.5\mathrm{k}_3+k_c \end{bmatrix}$$

Vibration Control of an Asymmetric Systems

$$\left(Ms^2+Cs+K+\sum_{i=1}^{j}\mu_i k_{ci}E_i\right)x(s)=p(s)+b(s)u(s)$$

Structural Modification

$$H_a(s)=\left(I+H(s)\sum_{i=1}^{j}\mu_i k_{ci}E_i\right)^{-1}H(s)$$

Active Control

$$\bar{H}(s)=H_a(s)-\frac{H_a(s)\,b\left(g+sf\right)^T H_a(s)}{1+\left(g+sf\right)^T H_a(s)\,b}$$

Partial Pole Placement of an Asymmetric system

Open-loop poles for $\mu = 0.5$

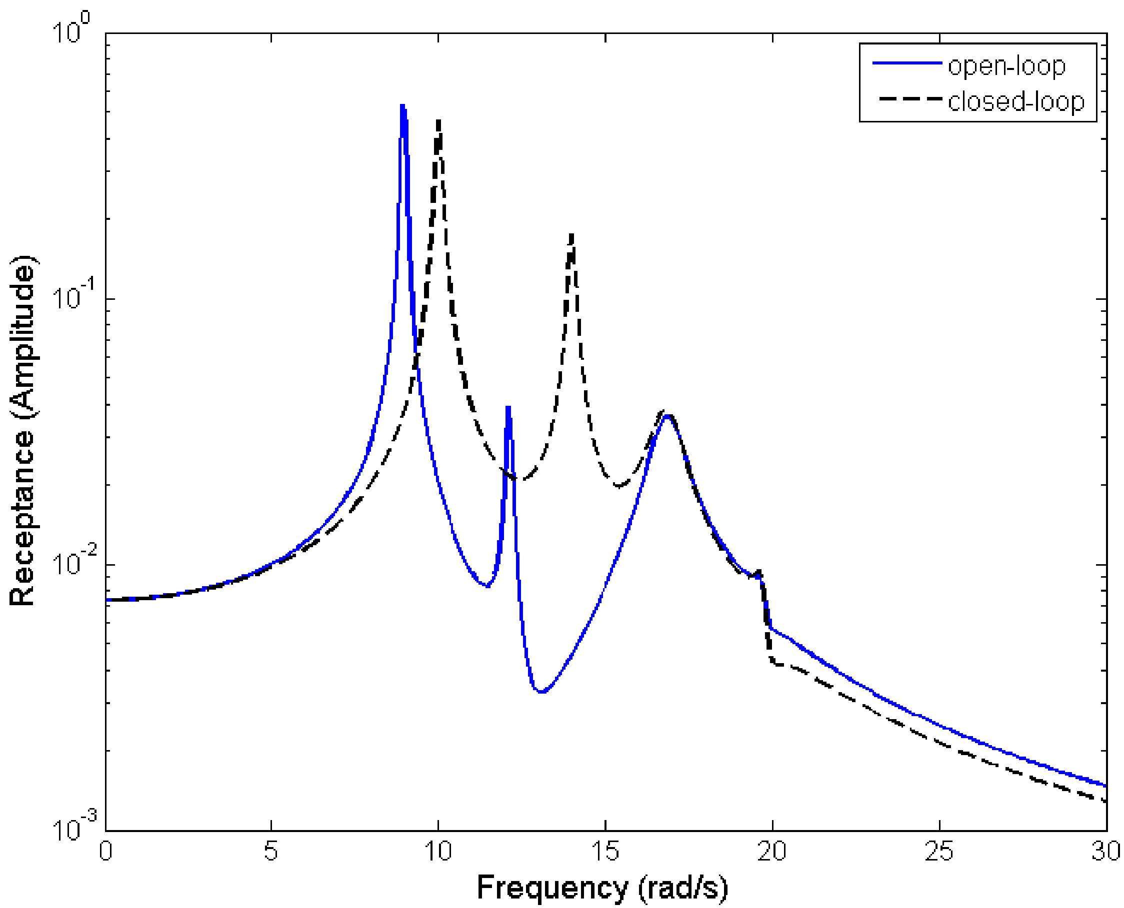
$$\lambda_{1,2} = 0.007 \pm 8.9462i$$

$$\lambda_{3,4} = -0.0553 \pm 12.1336i$$

Closed-loop poles

$$\mu_{1,2} = -0.08 \pm 10i$$

$$\mu_{3,4} = -0.1 \pm 14i$$



$$g = \begin{pmatrix} 0.7939 \\ -0.1655 \\ 1.0224 \\ 0.0771 \end{pmatrix} \qquad f = \begin{pmatrix} -0.7469 \\ 0.4484 \\ -0.8670 \\ 0.0665 \end{pmatrix}$$

Experimental Work?

Future work: Uncertain Dynamic Systems

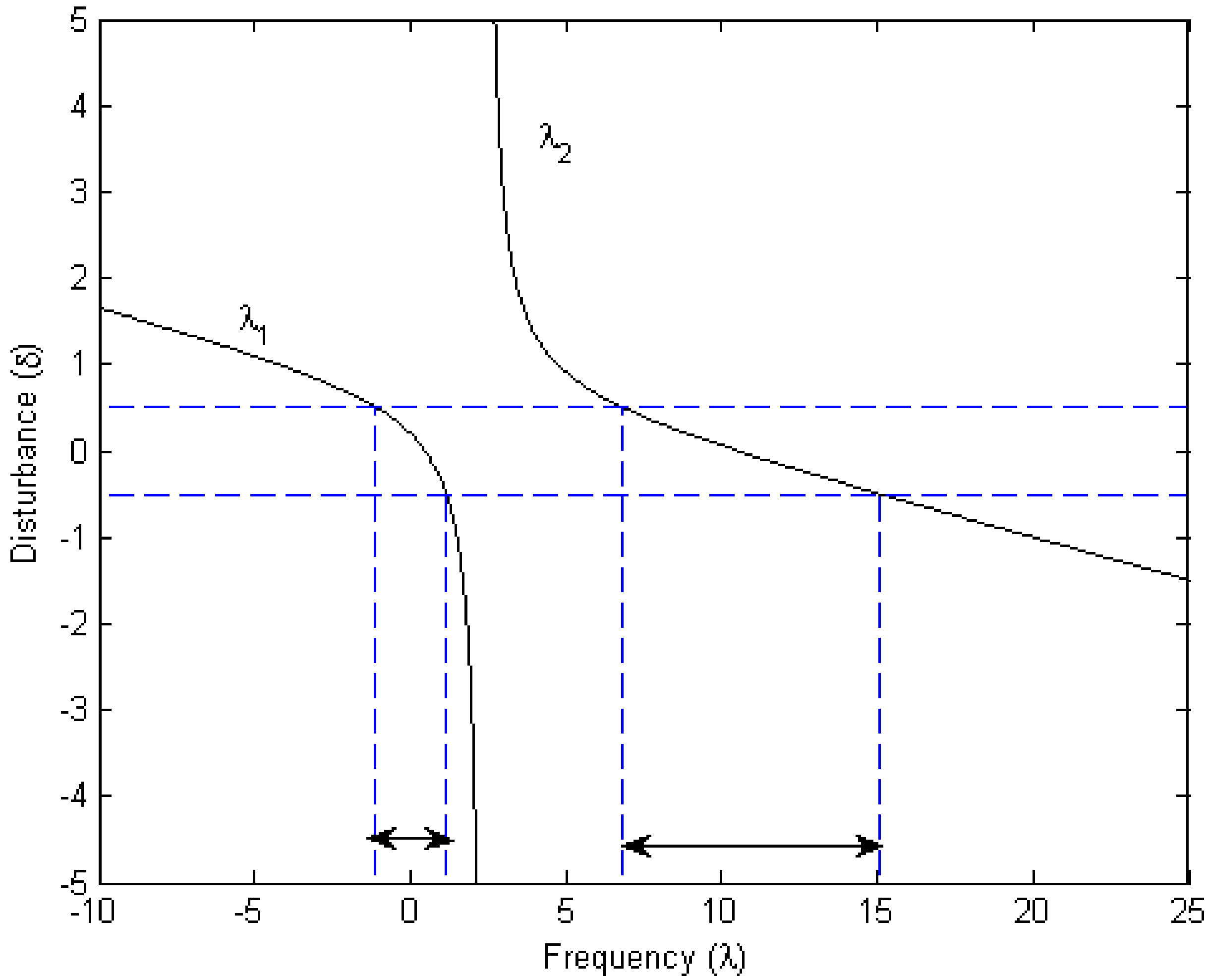
If we have a rank-1 disturbance to a dynamic system:

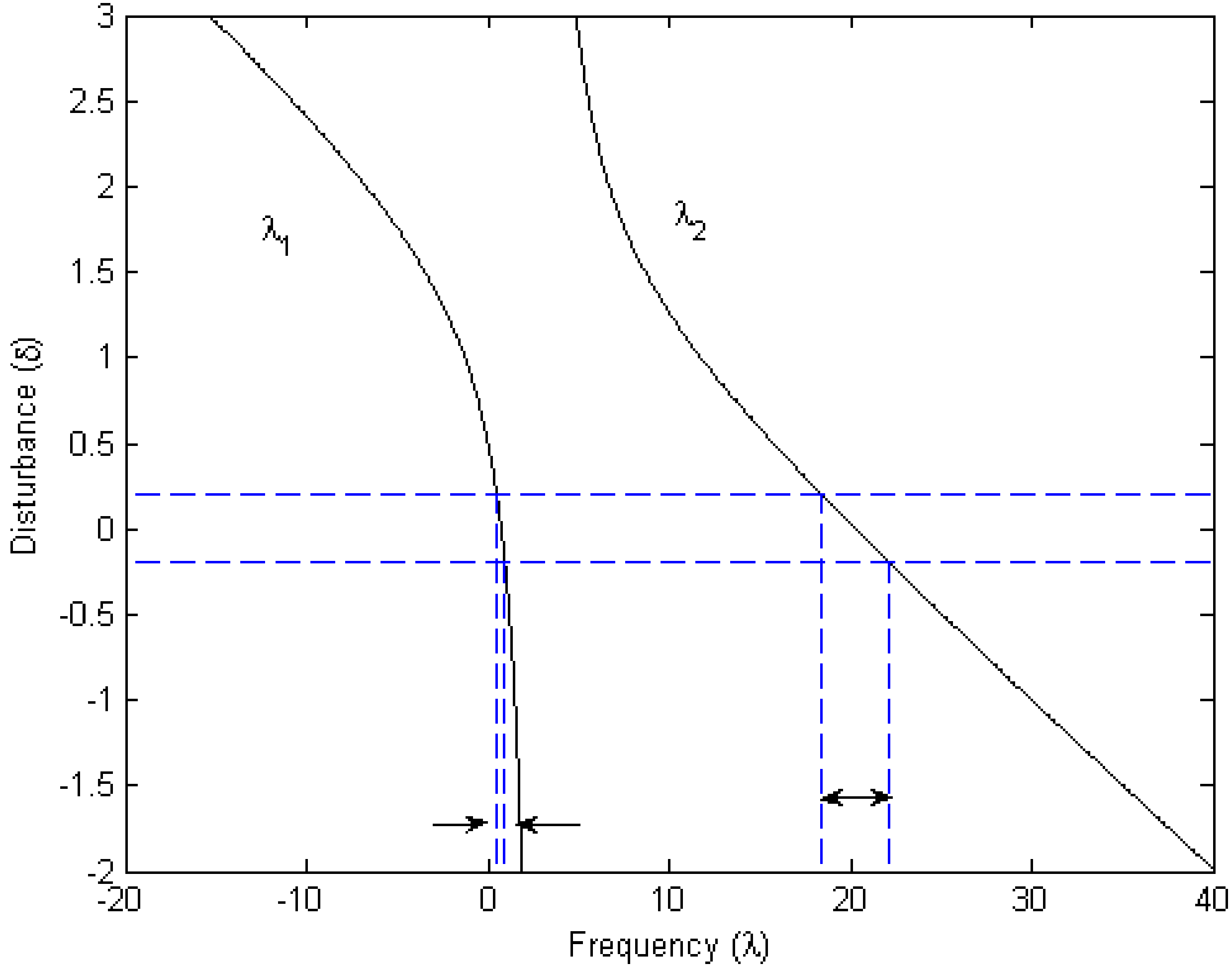
$$\hat{H}(s)=H(s)-\frac{H(s)b(g+sf)^T H(s)}{1+(g+sf)^T H(s)b}$$

Where are the open-loop poles of the system?

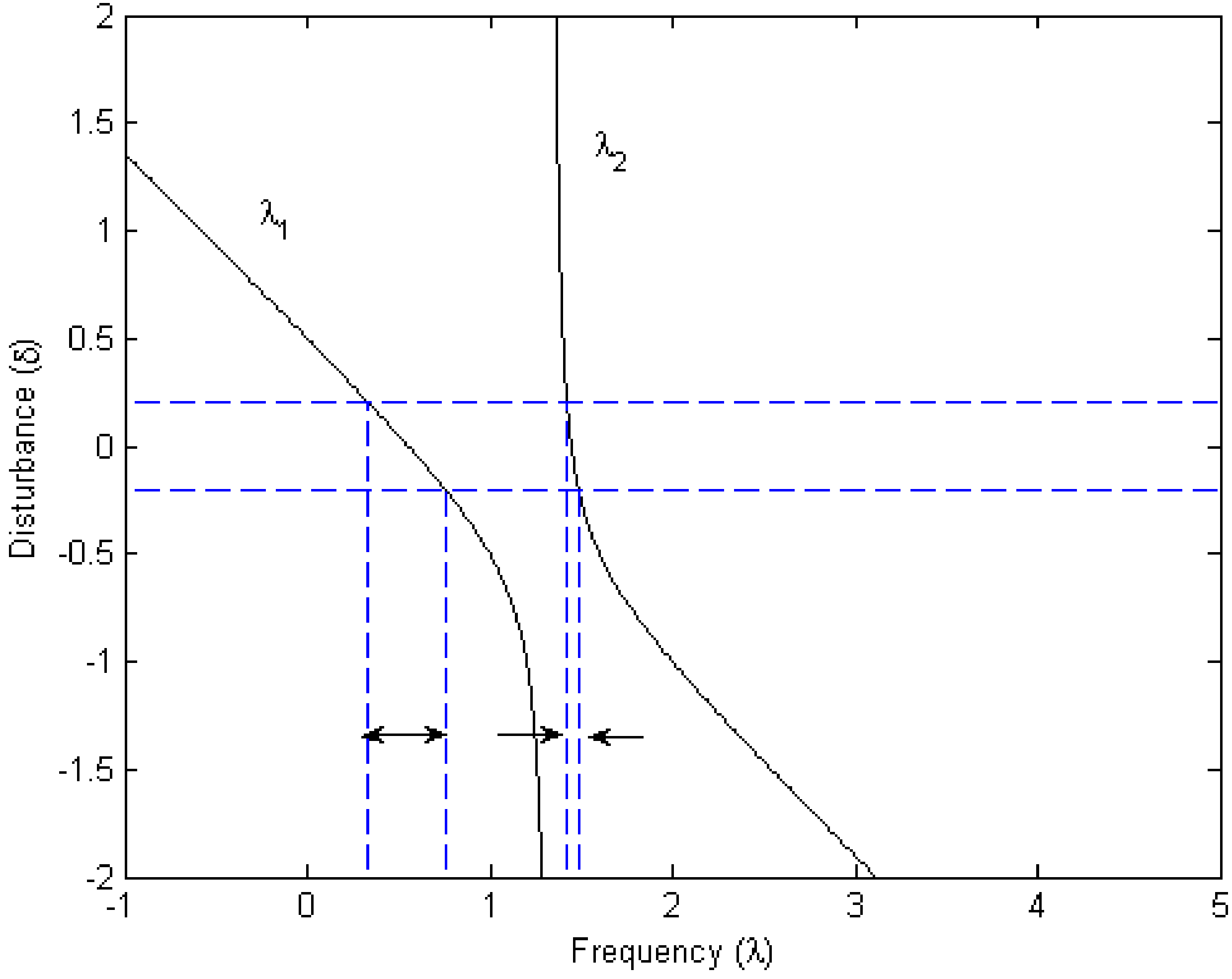
$$\delta(s)=\frac{-1}{d_r^T H(s)d_l}$$

Two degrees of freedom system





3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-20
-10
0
10
20
30
40
λ_1
λ_2
Disturbance (δ)
Frequency (λ)


λ1
λ2
Disturbance (δ)
Frequency (λ)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1
0
1
2
3
4
5

Stochastic Control

If we have a rank-1 disturbance to a dynamic system

$$\hat{H}(s) = H_c(s) - \frac{\delta(s) H_c(s) d_l d_r^T H_c(s)}{1 + \delta(s) d_r^T H_c(s) d_l}$$

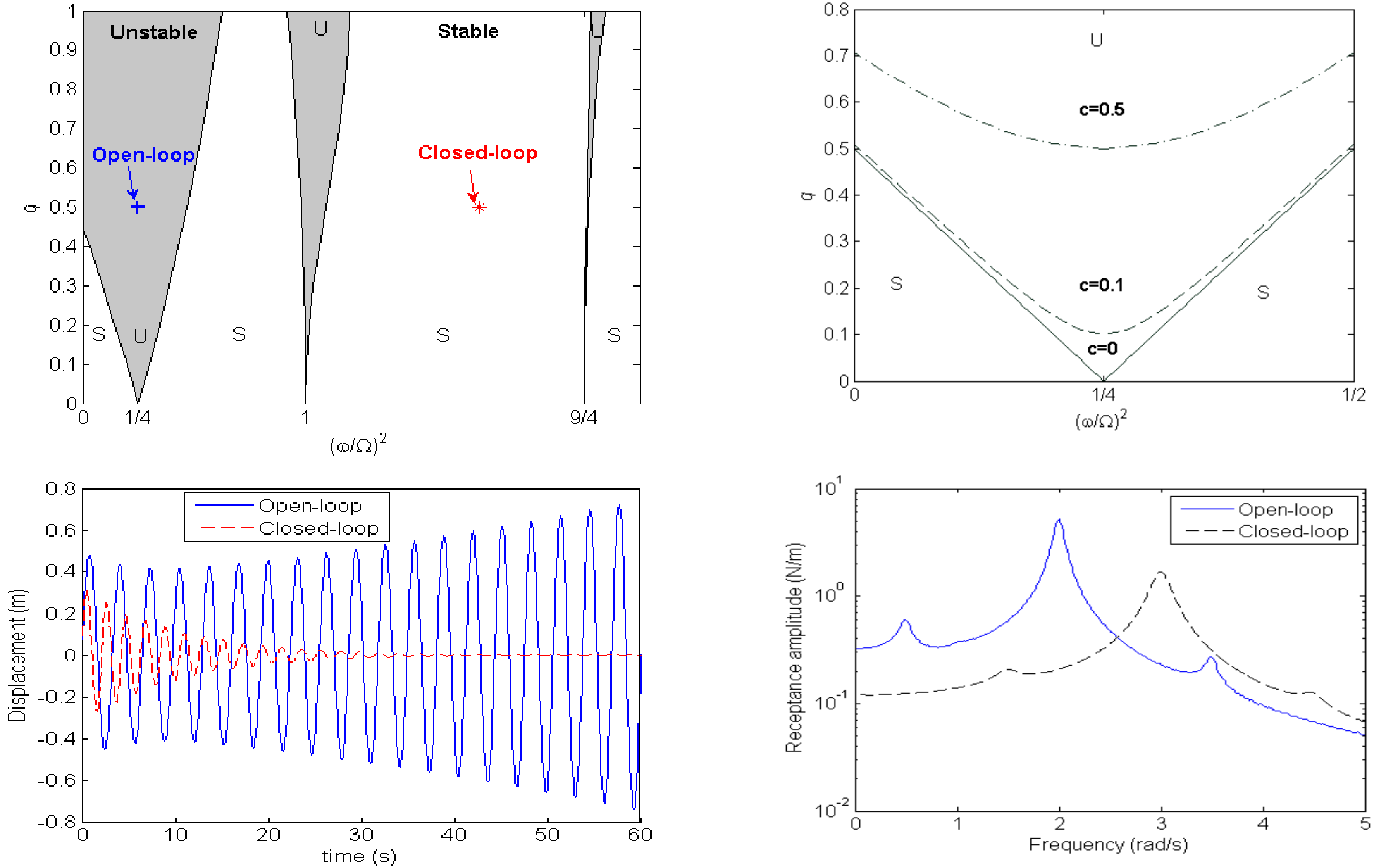
and a rank-1 control such as state feedback:

$$H_c(s) = H(s) - \frac{H(s) b \left(g + sf\right)^T H(s)}{1 + \left(g + sf\right)^T H(s) b}$$

Where are the closed-loop poles of the system?
Can we minimize the frequency length covered by the eigenvalues using the feedback control?

Future work: Parametric Excitation

$$\ddot{x}(t)+\left(\omega^2+q\cos\Omega t\right)x(t)=0$$



Conclusion

- The theory of the receptance method has been introduced and developed for:
  - Output feedback
  - Single-input state feedback
  - Sensitivity analysis
  - Partial pole placement
  - Robust pole placement

- There are numerous advantages over conventional matrix methods: no need to know or to evaluate the system matrices $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$; no need for an observer or for model reduction; no need to model the dynamic behaviour of actuators and sensors.

- Practical implementation of the receptance method on an Agusta-Westland W30 helicopter has been demonstrated.