Inverse Eigenvalue Problems in Control System Design

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Outline

1. Introduction - Overview

2. Inverse Eigenvalue Problem - Linear Systems

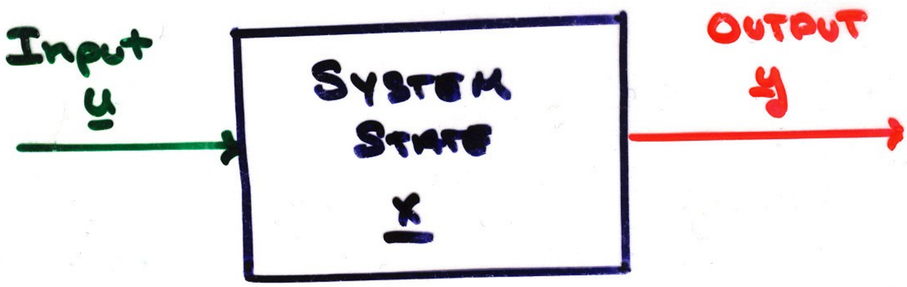
3. Inverse Problem - Quadratic Matrix Polynomial Systems

4. Conclusions / Future

1.

Introduction

DYNAMIC SYSTEM



Input $\underline{u}$ known / measurable

Output $\underline{y}$ known / measurable

State $\underline{x}$ NOT known / NOT measurable

PROPERTY

Given initial state $\underline{x}(t_0) = x_0$

and $\underline{u}(t)$ $\forall\, t \in [t_0, t^*]$

then $\underline{y}(t)$ is uniquely

determined $\forall\, t \in [t_0, t^*]$

Dynamical Models

Continuous	v	Discrete
- ODE/PDE		
Deterministic	v	Stochastic
Linear/Polynomial	v	Nonlinear
- DAE		
Time-Invariant	v	Non-autonomous

Dynamical Models

Continuous v Discrete

- ODE/PDE

Deterministic v Stochastic

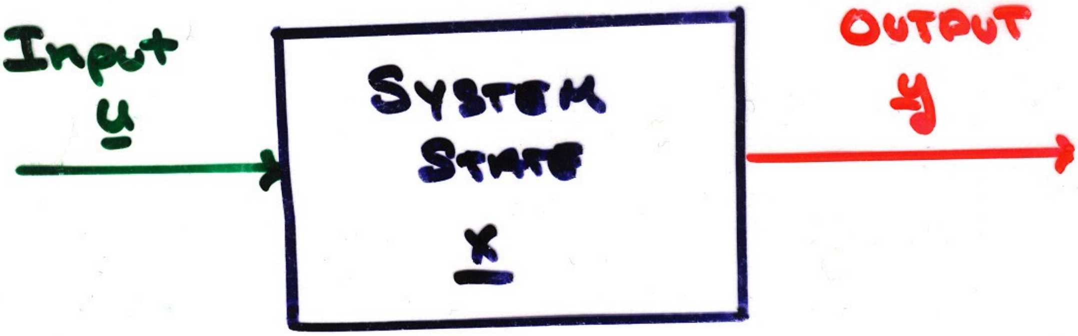
Linear/Polynomial v Nonlinear

- DAE

Time-Invariant v Non-autonomous

Automatic Feedback Control

DYNAMIC SYSTEM



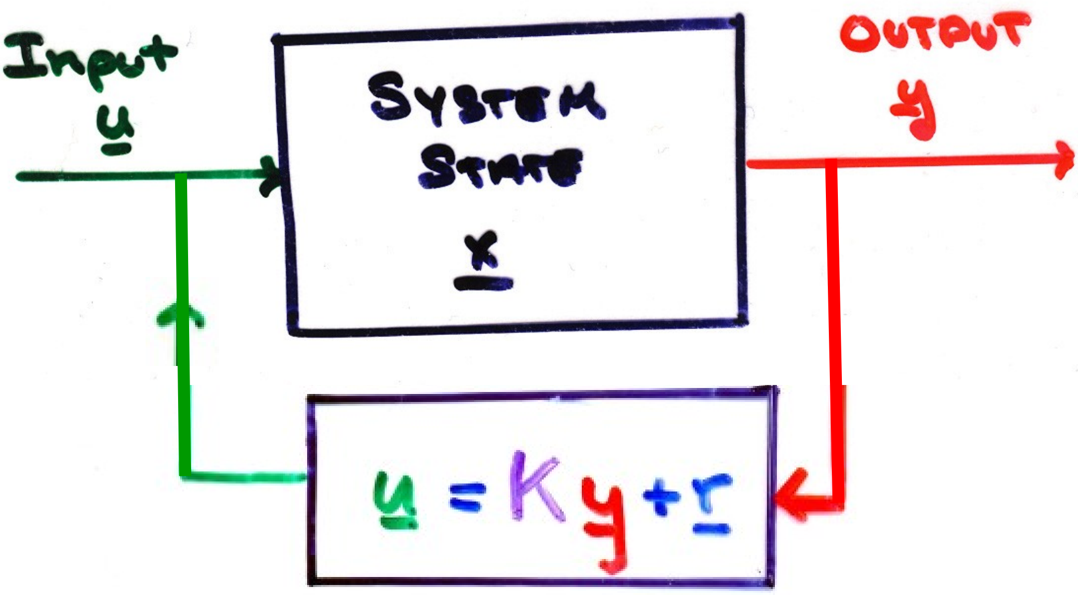
$\underline{x} \in \mathbb{R}^n$ $\quad \underline{u} \in \mathbb{R}^m$ $\quad \underline{y} \in \mathbb{R}^p$

SYSTEM:

$$\dot{\underline{x}} = A\underline{x} + B\underline{u}$$

$$\underline{y} = C\underline{x}$$

DYNAMIC SYSTEM



FEEDBACK SYSTEM:

$$\dot{\underline{x}} = A\underline{x} + BK\underline{y} + B\underline{r}$$

$$= (A + BKC)\underline{x} + B\underline{r}$$

$$\underline{y} = C\underline{x}$$

Objectives

Choose K to:

- assign eigenvalues / stabilize
- assign eigenvectors - decouple inputs/outputs
- ensure robustness (insensitivity to disturbances)

Applications

Aircraft Guidance and Control

Power Systems

Mechanical Industrial Plants

Acoustic Noise Control

Reservoir Flow Control

Chemical Reactors

Observers for State Estimation

…… …

2.

Inverse Eigenvalue Problem for LTI Systems

STATE FEEDBACK

System $\quad \dot{x} = Ax + Bu$ $\quad$ A $n \times n$, B $n \times m$

Feedback $\quad u = Kx + v \quad \Rightarrow$

$$\dot{x} = (A + BK)x + Bv$$

PROBLEM Find real K s.t

$A+BK$ has given eigenvalues

$$\mathcal{L} = \{\lambda_i \mid \lambda_i \in \mathbb{C},\ \lambda_i \in \mathcal{L} \Rightarrow \bar{\lambda}_i \in \mathcal{L}\}$$

Existence $\exists K$ for every set

$\mathcal{L}$ if and only if (A, B)

is controllable ie

iff $\operatorname{rank}[B, A - \lambda I] = n$

$\forall \lambda \in \mathbb{C}$

THEOREM

Given Λ, X non-singular

$\exists\, K$ satisfying $(A+BK)X = X\Lambda$

iff $\quad U_1^T (AX - X\Lambda) = 0,$

where $\quad B = [U_0, U_1] \begin{bmatrix} Z \\ 0 \end{bmatrix}.$

Then

$$K = Z^{-1} U_0^T (X\Lambda X^{-1} - A)$$

Corollary

$$x_j \in \mathcal{S}_j \equiv \mathcal{N}\{U_1^T (A - \lambda_j I)\}$$

Proof:

$$(A + BK)X = X\Lambda$$

implies

$$BK = X\Lambda X^{-1} - A$$

Using decomposition of B gives

$$\left.\begin{aligned} ZK &= U_0^{T}(X\Lambda X^{-1} - A) \\ 0 &= U_1^{T}(X\Lambda X^{-1} - A) \end{aligned}\right\}$$

which establishes the theorem

Proof:

$$(A + BK)X = X\Lambda$$

implies

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which establishes the theorem

Solution is NOT UNIQUE if m > 1

SENSITIVITY (ROBUSTNESS)

Choose K s.t. λ_i are insensitive to perturbations in M

MEASURE OF SENSITIVITY

The first order change $\delta\lambda$ in pole λ due to a perturbation Δ in M is given by

$$\delta\lambda = \underline{y}^T \Delta \underline{x} \,/\, \underline{y}^T \underline{x}$$

where

$$M x_j = \lambda_j x_j \,, \quad y_j^T M = \lambda_j y_j^T \,,$$

$M = A + BK$ non-defective

UNSTRUCTURED PERTURBATIONS

If perturbations are entirely arbitrary, then worst case occurs when

$$\Delta = \frac{y \underline{x}^T}{\|y\| \, \|\underline{x}\|} \gamma$$

Then

$$|\delta\lambda| = \frac{\|y\| \, \|\underline{x}\|}{|y^T \underline{x}|} |\gamma| = c(\lambda) \, \|\Delta\|$$

Here

$$c(\lambda) = \|y\| \, \|\underline{x}\| \,/\, |y^T \underline{x}|$$

is the sensitivity coefficient of λ.

Robustness - Unstructured Case

As a global measure of sensitivity we take

$$\nu^2 = \sum_j c(\lambda_j)^2 = \|X^{-1}\|_F^2$$

where $\underline{y}_j^T \underline{x}_j = 1$, $\|\underline{x}_j\| = 1$

$$Y^T = \begin{bmatrix} \underline{y}_1^T \\ \underline{y}_2^T \\ \vdots \\ \underline{y}_n^T \end{bmatrix} = [\underline{x}_1, \underline{x}_2 \ldots \underline{x}_n]^{-1} = X^{-1}$$

Remark Techniques for minimizing ν are given in

Kautsky, Nichols and Van Dooren [KNVD]
I. J. C., 1985

NUMERICAL METHOD

STEP A : Find bases for $\mathcal{S}_j$.
Select initial X s.t
$x_j = X e_j \in \mathcal{S}_j$

STEP X : Update each vector iteratively by solving

$$\min_{x_j \in \mathcal{S}_j} \nu_F(X)$$

subject to $\|x_j\| = 1$.

STEP K : Find M s.t.
$MX = X\Lambda$ and contruct
$K = Z^{-1} U_0^T (M - A)$

Extensions

Structured Perturbations

Differential-Algebraic /
Singular Descriptor Systems

Partial Eigenvalue Assignment

Output Feedback

Observer / State Estimation

Quadratic Systems

3.

Inverse Eigenvalue Problems for Quadratic Matrix Polynomial Systems

Second Order Control System

System:

$$\ddot{\underline{z}} - A_2\dot{\underline{z}} - A_1\underline{z} = B\underline{u}$$

Feedback:

$$\underline{u} = K_1\underline{z} + K_2\dot{\underline{z}} + \underline{r} \quad \Rightarrow$$

Closed Loop System:

$$\ddot{\underline{z}} - M_2\dot{\underline{z}} - M_1\underline{z} = B\underline{r},$$

where

$$M_1 = A_1 + BK_1, \quad M_2 = A_2 + BK_2$$

Inverse Problem

Find K_1 K_2 s.t.
closed loop pencil has
specified eigenvalues $\{\lambda_i\}_1^{2n}$
– i.e. s.t.

$$\det(\lambda_i^2 I - \lambda_i M_2 - M_1) = 0$$

and s.t. assigned λ_i
are **Robust** = insensitive
to perturbations in
M_1 and M_2

EXISTENCE

Solutions K_1 K_2 exist for every self-conjugate set $\{\lambda_i\}_1^{2n}$ if and only if the system is

completely controllable

that is:

$$\operatorname{rank}[\lambda^2 I - \lambda A_2 - A_1, B] = n$$

$$\forall \lambda \in \mathbb{C}.$$

APPLICATIONS

- Large Flexible Space Structures
- Vibration in Multi-Body Systems
- Damped Gyroscopic Systems
- Robotics
- Optimal Quadratic Control

Sensitivity

Perturbations $\delta M_1, \delta M_2$ in $M_1, M_2 \Rightarrow$

$$|\delta\lambda| \leq c(\lambda) \, \| [\delta M_1, \delta M_2] \|$$

where

$$c(\lambda) = \frac{\|\underline{w}^H\| \, \|\underline{v}\| \, (1 + |\lambda|^2)^{1/2}}{|\underline{w}^H (2\lambda I - M_2) \underline{v}|}$$

and

$$(\lambda^2 I - \lambda M_2 - M_1)\underline{v} = 0$$

$$\underline{w}^H (\lambda^2 I - \lambda M_2 - M_1) = 0$$

Robustness Measure

$$\nu^2 = \sum_1^{2n} c^2(\lambda_j) = \sum_1^{2n} \| \underline{w}_j^H \|_2^2$$

where

- λ_j non-defective
- $(1 + |\lambda_j|^2)^{1/2} \| \underline{v}_j \|_2 = 1$
- $| \underline{w}_j^H (2\lambda_j I - M_2) \underline{v}_j | = 1$

(λ_j multiple - assume $\{v_j\}$ $\{w_j^H\}$ are biorthogonal bases for invariant subspaces)

EMBEDDING APPROACH

SYSTEM: $\quad \dot{\underline{x}} = \hat{A}\underline{x} + \tilde{B}\underline{u}$

$$\underline{x} = \begin{bmatrix} \underline{z} \\ \dot{\underline{z}} \end{bmatrix} \quad \hat{A} = \begin{bmatrix} 0 & I \\ A_1 & A_2 \end{bmatrix} \quad \tilde{B} = \begin{bmatrix} 0 \\ B \end{bmatrix}$$

FEEDBACK:

$$\underline{u} = \tilde{K}\underline{x} + \underline{r} = [K_1 \; K_2]\begin{bmatrix} \underline{z} \\ \dot{\underline{z}} \end{bmatrix} + \underline{r}$$

CLOSED LOOP: $\quad \dot{\underline{x}} = \tilde{M}\underline{x} + \tilde{B}\underline{r}$

$$\tilde{M} = \begin{bmatrix} 0 & I \\ M_1 & M_2 \end{bmatrix}$$

$$M_1 = A_1 + BK_1 \qquad M_2 = A_2 + BK_2$$

Inverse Linear Problem

Find $\tilde{K} = [K_1, K_2]$ s.t.

closed loop matrix $\tilde{M}$ has specified eigenvalues $\{\lambda_i\}_1^{2n}$

- ie s.t

$$\det(\lambda_i I - \tilde{M}) = 0$$

and s.t. assigned λ_i are Robust = insensitive to perturbations in $\tilde{M}$.

METHOD

Apply known methods for robust eigenstructure assignment to the linear problem:

KAUTSKY, NICHOLS & VAN DOOREN, INT. J. CONTROL (1985)

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$$\min C(\lambda) = \frac{\|\underline{\tilde{w}}\| \, \|\underline{\tilde{v}}\|}{|\underline{\tilde{w}}^H \underline{\tilde{v}}|}$$

~ WORST CASE perturbation

DIFFICULTIES

- Linear theory does not take into account the **STRUCTURE** of the allowable perturbations:

$$\delta \tilde{M} = \begin{bmatrix} 0 & 0 \\ \delta M_1 & \delta M_2 \end{bmatrix}$$

- Procedure does not take into account special form of **EIGENSTRUCTURE**:

$$\underline{\tilde{v}} = \begin{bmatrix} \underline{v} \\ \lambda \underline{v} \end{bmatrix} \qquad \underline{\tilde{w}}^H = \begin{bmatrix} \lambda \underline{w}^H - \underline{w}^H M_2 , & \underline{w}^H \end{bmatrix}$$

Sensitivity to Structured Perturbations

Perturbation $\delta\tilde{M} = F\Delta G^T$ in $\tilde{M}$ where Δ is unknown (arbitrary) and F, G given

$\Rightarrow$

$$|\delta\lambda| = \frac{|\tilde{\underline{w}}^T \delta\tilde{M}\hat{\underline{v}}|}{|\tilde{\underline{w}}^T\hat{\underline{v}}|} \leq \tilde{c}(\lambda)\|\Delta\|$$

where

$$\tilde{c}(\lambda) = \frac{\|\tilde{\underline{w}}^T F\| \; \|G^T\hat{\underline{v}}\|}{|\tilde{\underline{w}}^T \hat{\underline{v}}|}$$

and

$$(\lambda I - \tilde{M})\hat{\underline{v}} = 0 \qquad \tilde{\underline{w}}^T(\lambda I - \tilde{M}) = 0$$

APPLICATION TO QUADRATIC PROBLEM

Set : $F = \begin{bmatrix} 0 \\ I \end{bmatrix} \quad G^T = I$

Then :

$$\delta\tilde{M} = F\,[\delta M_1 \;\; \delta M_2]\,G^T = \begin{bmatrix} 0 & 0 \\ \delta M_1 & \delta M_2 \end{bmatrix}$$

is allowable perturbation

and sensitivity is :

$$\tilde{c}(\lambda) = \|\underline{\tilde{w}}^H F\| \, \|G^T \underline{\tilde{v}}\| \,/\, |\underline{\tilde{w}}^H \underline{\tilde{v}}|$$

$$= \frac{\|\underline{w}^H\| \, \|v\| \, (1+|\lambda|^2)^{1/2}}{|\underline{w}^H (2\lambda I - M_2)\underline{v}|} \equiv c(\lambda)$$

using

$$\underline{\tilde{w}}^H = [\lambda \underline{w}^H - \underline{w}^H M_2, \; \underline{w}^H], \quad \underline{\tilde{v}} = \begin{bmatrix} \underline{v} \\ \lambda \underline{v} \end{bmatrix}$$

HENCE:

Can solve Quadratic problem using techniques for robust eigenstructure assignment in Linear systems subject to structured perturbations:

KAUTSKY + NICHOLS
Systems + Control Letters (1990)

MODIFIED THEORY

(i) Let $B = [U_0, U_1]\begin{bmatrix} Z_B \\ 0 \end{bmatrix}$

$U = [U_0\ U_1]$ orthogonal

Z_B non-singular

Then for prescribed λ_j the vector $\underline{v}_j$ satisfies

$$(\lambda_j^2 I - \lambda_j M_2 - M_1)\,\underline{v}_j = 0$$

if and only if

$$\underline{v}_j \in \mathcal{S}_j = \mathcal{N}\{U_1^T(\lambda_j^2 I - \lambda_j A_2 - A_1)\}$$

(ii) Let

$$[K_1, K_2] = Z_B^{-1} U_0^T (V\Lambda^2 - A_2 V\Lambda - A_1 V)\tilde{V}^{-1}$$

$$= Z_B^{-1} U_0^T (V\Lambda^3 W^H - (V\Lambda^2 W^H)^2 - A_1 ,$$

$$V\Lambda^2 W^H - A_2)$$

where

$V = [\underline{v}_1, \underline{v}_2 \dots \underline{v}_{2n}]$ with $\underline{v}_j \in \mathcal{S}_j$

and

$\Lambda = \mathrm{diag}\{\lambda_j\}$, $\tilde{V} = \begin{bmatrix} V \\ V\Lambda \end{bmatrix}$ $W^H = \tilde{V}^{-1} \begin{bmatrix} 0 \\ I \end{bmatrix}$

Then the closed loop system has the prescribed eigenvalues $\{\lambda_j\}$

NUMERICAL ALGORITHM

STEP 1: Select initial set

$V = [\underline{v}_1, \underline{v}_2, \cdots \underline{v}_{2n}]$, $\underline{v}_j \in \mathcal{S}_j$

s.t. $\tilde{V} = \begin{bmatrix} V \\ V\Lambda \end{bmatrix}$ is nonsingular

STEP 2: Update each $\underline{v}_j$

by minimizing robustness:

$$\min_{\underline{v}_j \in \mathcal{S}_j} \sum_j c^2(\lambda_j) \equiv \left\| \tilde{V}^{-1} \begin{bmatrix} 0 \\ I \end{bmatrix} \right\|_F^2 = \|W^H\|_F^2$$

subject to $(1+|\lambda_j|^2)^{1/2} \|\underline{v}_j\| = 1$

STEP 3: Construct K_1, K_2 from

V and W as in Theorem.

Step 1/3

Can always be achieved if the system is

completely controllable

that is :

$$\operatorname{rank}\left[\lambda^2 I - \lambda A_2 - A_1, B\right] = n$$

and $\{\lambda_i\}$ are distinct

(or satisfy other mild restrictions)

KEY STEP = STEP 2

- Iterative
- Each iteration requires
  1 QR update $(2n \times (2n-1))$
  1 Householder $(2n \times 1)$
  1 least square solve $(2n \times (m-1))$, $m \ll n$
- $\tilde{V}^{-1}$ guaranteed nonsingular after each update
- FEW iterations needed to obtain robust solution

EXTENSION : GENERALIZED CASE

SYSTEM :

$$J\underline{\ddot{z}} + D\underline{\dot{z}} + C\underline{z} = B\underline{u}$$

EMBEDDING :

$$\begin{bmatrix} I & 0 \\ 0 & J \end{bmatrix} \begin{bmatrix} \underline{\dot{z}} \\ \underline{\ddot{z}} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -C & D \end{bmatrix} \begin{bmatrix} \underline{z} \\ \underline{\dot{z}} \end{bmatrix} + \begin{bmatrix} 0 \\ B \end{bmatrix} \underline{u}$$

$$\Rightarrow \quad \tilde{E}\underline{\dot{x}} = \tilde{A}\underline{x} + \tilde{B}\underline{u}$$

INVERSE PROBLEM

Find K_1 K_2 to assign eigenvalues $\{\lambda_i\}_1^p$ s.t.

$$\det(\lambda_i^2 J - \lambda_i M_2 - M_1) = 0$$

$$M_1 = C + BK_1 \qquad M_2 = D + BK_2$$

and s.t. assigned λ_i are ROBUST = insensitive to perturbations in

J, M_1 and M_2

ANALYSIS - J NONSINGULAR

Perturbations $J\delta J$ $J\delta M_1$ $J\delta M_2$ in $J, M_1, M_2 \Rightarrow$

$$|\delta\lambda| \le c(\lambda)\, \|[\delta J, \delta M_1, \delta M_2]\|_2$$

where

$$c(\lambda) = \frac{\alpha \|\underline{w}^H J\|_2 \|\underline{v}\|_2}{|\underline{w}^H (2\lambda J - D)\underline{v}|}$$

$$\alpha = \left(1 + |\lambda|^2 + |\lambda|^4\right)^{1/2}$$

and

$$(\lambda^2 J - \lambda M_2 - M_1)\underline{v} = 0$$

$$\underline{w}^H(\lambda^2 J - \lambda M_2 - M_1) = 0$$

Robustness Measure

$$\nu^2 = \sum_{1}^{2n} c^2(\lambda_j) = \sum_{1}^{2n} \| \underline{w}_j^H J \|_2^2$$

where

- λ_j non-defective
- $(1 + |\lambda_j|^2 + |\lambda_j^4|)^{1/2} \, \| \underline{v}_j \|_2 = 1$
- $| \underline{w}_j^H (2 \lambda_j J - D) \underline{v}_j | = 1$

(for multiple λ_j assumes v_j, w_j^H form biorthogonal bases for invariant subspaces)

EXAMPLE 1

$$J = 10 I_3 \qquad D = 0$$

$$C = \begin{bmatrix} -40 & 40 & 0 \\ 40 & -80 & 40 \\ 0 & 40 & -40 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 \\ 3 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\mathcal{L} = \{\pm 3.6039i,\ \pm 2.4940i,\ \pm 0.89008i\}$$

Assign: $\mathcal{L} = \{-1, -2, -3, -4, -5, -6\}$

Initial guess:

$\nu = 2808$ $\qquad \kappa(\tilde{V}) = 3438$

First iteration:

$\nu = 137.6$

Third iteration:

$\nu = 136.6$ $\qquad \kappa(\tilde{V}) = 497.2$

$$K_1 = \begin{bmatrix} -1.2566 & -44.622 & 120.23 \\ 56.184 & 42.276 & -227.72 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 86.175 & -27.228 & -16.522 \\ -85.494 & 13.016 & -4.5524 \end{bmatrix}$$

Perturbations :

$$\| [\delta M_1, \delta M_2] \|_2 \le \varepsilon = 0.002$$

$$\Rightarrow \quad |\delta\lambda| = O(0.01)$$

Corresponds to absolute perturbations

$$J\delta M_1, \; J\delta M_2 = O(0.01)$$

$\Rightarrow$ SYSTEM ROBUST

EXAMPLE 2

D, C, B as in Example 1

$$J = \begin{bmatrix} 5000 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1.00001 \end{bmatrix}$$

NEARLY SINGULAR !

$$\mathcal{L} = \{\pm 4999.0i, \pm 4.4726i, \pm 0.051653i\}$$

Assign: $\mathcal{L} = \{-1, -2, -3, -4, -5, -6\}$

Initial guess:

$\nu = 4.47_{10}5$ $\quad\kappa(\tilde{V}) = 5.24_{10}5$

First iteration:

$\nu = 3.298_{10}4$

Third iteration:

$\nu = 3.156_{10}4$ $\quad\kappa(\tilde{V}) = 7.622_{10}4$

$$K_1 = \begin{bmatrix} 2668.9 & 59.004 & -58.413 \\ 19.996 & -60.000 & 60.000 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 1291.4 & -3.2463 & -3.1778 \\ -0.018680 & 0.45542_{10}{-5} & -0.53043_{10}{-4} \end{bmatrix}$$

Perturbations :

$$\| [\delta J, \delta M_2, \delta M_1] \|_2 \leq \varepsilon = 3_{10}{-6}$$

$$\Rightarrow \quad |\delta\lambda| = O(0.01)$$

Corresponds to absolute errors

$$J\delta J, \; J\delta M_2, \; J\delta M_1 = O(0.01)$$

$\Rightarrow$ ROBUST CLOSED LOOP SYSTEM

despite ill-conditioning of J !

ANALYSIS - SINGULAR CASE

Results can be extended to general case where quadratic polynomial is SINGULAR provided system is STRONGLY CONTROLLABLE.

Sensitivity analysis uses CHORDAL measure

cf. KAUTSKY + NICHOLS, LAA (1989)

CONCLUSIONS

- QUADRATIC inverse eigenvalue problem can be treated by using structured perturbation theory for the embedded LINEAR problem

- Efficiency can be improved by exploiting special eigenstructure of Quadratic System

- Technique can be extended to Differential-Algebraic Systems

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